SUMMARY OF THE DERIVATIVE

NOTATIONS FOR THE DERIVATIVE: $y = f(x)$

Order	Variable-Prime	Function-Prime	Leibniz	Explicit Euler	Implicit Euler
1^{st}	y'	$f'(x)$	$\frac{dy}{dx}$	$D_x f$	Df
2^{nd}	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$D_x^2 f$	$D^2 f$
3^{rd}	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$D_x^3 f$	$D^3 f$
4^{th}	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$D_x^4 f$	$D^4 f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n^{th}	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$D_x^n f$	$D^n f$

DEFINITION OF THE DERIVATIVE OF $f(x)$: $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

INTERPRETATION OF THE DERIVATIVE:
Geometrically, $f'(x_0)$ means the slope of the **tangent** line to curve $f(x)$ at the point $x = x_0$.

In applications, 'derivative' is synonymous with '**instantaneous** rate of change'.
(e.g. If $s(t)$ measures distance over time, then $s'(t_0)$ is the instantaneous speed at time t_0.)

In business/finance/economics, the word 'marginal' means 'derivative'.
(e.g. If $P(x)$ measures the total profit gained after x items are sold,
then $P'(x)$ is the actual profit gained when the $(x+1)^{st}$ item is sold.)

DERIVATIVE RULES:

Constant Rule: $f(x) = k \Rightarrow f'(x) = 0$ [k is a real number]

Power Rule: $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$ [n is a real number]

Constant Multiple Rule: $f(x) = c\,g(x) \Rightarrow f'(x) = c\,g'(x)$ [c is a real number]

Sum/Difference Rule: $f(x) = g(x) \pm h(x) \Rightarrow f'(x) = g'(x) \pm h'(x)$

Product Rule: $f(x) = g(x)h(x) \Rightarrow f'(x) = g'(x)h(x) + g(x)h'(x)$

Quotient Rule: $f(x) = \dfrac{g(x)}{h(x)} \Rightarrow f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

Chain Rule (Usual): $f(x) = (g \circ h)(x) = g[h(x)] \Rightarrow f'(x) = g'[h(x)]h'(x)$

Chain Rule (Leibniz): $v = f(u), u = g(x) \Rightarrow \dfrac{dv}{dx} = \dfrac{dv}{du}\dfrac{du}{dx}$

Natural Exponential Rule: $f(x) = e^x \Rightarrow f'(x) = e^x$ [$e \approx 2.7183$ is the natural logarithm base]

Natural Logarithm Rule: $f(x) = \ln x \Rightarrow f'(x) = \dfrac{1}{x} = x^{-1}$

Exponential Rule: $f(x) = a^x \Rightarrow f'(x) = a^x(\ln a)$ [a is a positive real number]

Logarithm Rule: $f(x) = \log_a x \Rightarrow f'(x) = \dfrac{1}{x \ln a}$ [a is a positive real number]

References

[1] S. Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences*. Brooks Cole, Belmont, CA, 5th Edition, 2008.