SUMMARY OF THE INDEFINITE INTEGRAL

NOTATION FOR THE INDEFINITE INTEGRAL (AKA ANTI-DERIVATIVE):

$\int f(x)\,dx$ reads "The integral of $f(x)$ with respect to x"

Here, x is called a **dummy variable** – any variable can be used since it just acts as a **label**:

For example, $\int x^2\,dx = \int y^2\,dy = \int z^2\,dz = \int w^2\,dw = \cdots$

DEFINITION OF THE INTEGRAL OF $f(x)$: $F(x) = \int f(x)\,dx \iff F'(x) = f(x)$

INTERPRETATION OF THE INTEGRAL:

The integral of $f(x)$ is the **inverse operation** of the derivative of $f(x)$.
Therefore, the integral of $f(x)$ is a function plus an **arbitrary constant** C: $\int f(x)\,dx = F(x) + C$

Geometrically, $\int f(x)\,dx$ is a **family of curves** $F(x) + C$ where the slope of $F(x)$ equals $f(x)$ at each x.

INTEGRAL RULES:

Constant Rule: $\int k\,dx = kx + C$ [k is a real number]

Power Rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ [$n \neq -1$ is a real number]

Constant Multiple Rule: $\int kf(x)\,dx = k\int f(x)\,dx$ [k is a real number]

Sum/Difference Rule: $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Natural Exponential Rule: $\int e^x\,dx = e^x + C$ [$e \approx 2.7183$ is the natural logarithm base]

Natural Logarithm Rule: $\int \frac{1}{x}\,dx = \ln|x| + C$

Exponential Rule: $\int a^x\,dx = \frac{a^x}{\ln a} + C$ [$a \neq 1$ is a positive real number]

REMARKS:

Functions of arbitrary constants yield arbitrary constants. (e.g. $C_1 + C_2 = C_3,\ \sqrt{C_1} = C_4,\ e^{C_1} = C_5, \ldots$)
More complicated integrals often can be evaluated using the **Substitution Method**.
There are no "product or quotient rules" for integrals.
There are integrals that cannot be represented by simple formulas – such integrals are called **nonelementary**.

Here are some nonelementary integrals:

$$\int e^{x^2}\,dx,\quad \int e^{e^x}\,dx,\quad \int \frac{e^x}{x}\,dx,\quad \int \sqrt{1+x^4}\,dx,\quad \int \ln(\ln x)\,dx,\quad \int \frac{1}{\ln x}\,dx,\quad \int x^x\,dx$$

These shortcomings of integration are typically addressed in a 2^{nd} semester calculus course (MATH 1352).

References

[1] S. Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences*. Brooks Cole, Belmont, CA, 5th Edition, 2008.