INFLECTION POINTS & CONCAVITY OF FUNCTIONS

DEFINITIONS:

$f''(x) > 0 \iff f(x)$ is **concave up** ('smiles') at point x
$f''(x) < 0 \iff f(x)$ is **concave down** ('frowns') at point x

$f(x)$ has an inflection point at $x \iff$ the concavity of $f(x)$ changes at x.
Inflection points can only **possibly** occur where $f''(x) = 0$ or $f''(x)$ does not exist (DNE).

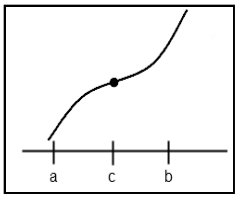
INTERPRETATION OF THE 2^{nd} DERIVATIVE (CONCAVITY):

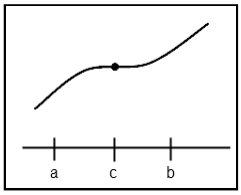
In applications, **2^{nd} derivative** is synonymous with **instantaneous rate of rate of change**.
(e.g. If $s(t)$ measures distance over time, then $s''(t_0)$ is the instantaneous acceleration at time t_0.)

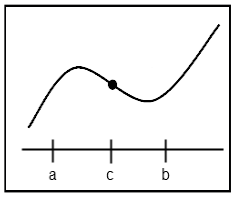
If $P(x)$ measures the total profit gained after x items are sold, then:
$P'(x) > 0$ & $P''(x) > 0$ means profit is increasing **more and more**.
$P'(x) > 0$ & $P''(x) < 0$ means profit is increasing **less and less**.
$P'(x) < 0$ & $P''(x) < 0$ means profit is decreasing **more and more**.
$P'(x) < 0$ & $P''(x) > 0$ means profit is decreasing **less and less**.

CASE I:

x	a	c	b
$f''(x)$	$-$	0	$+$
concavity	$\cap$	$*$	$\cup$

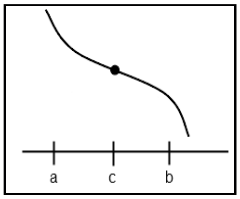


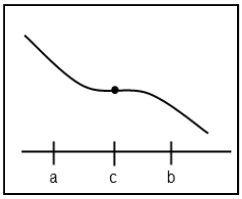


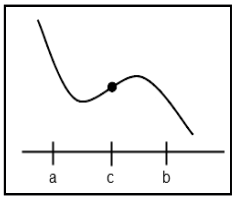


CASE II:

x	a	c	b
$f''(x)$	$+$	0	$-$
concavity	$\cup$	$*$	$\cap$

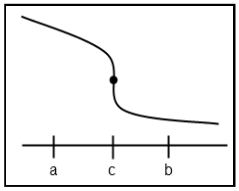


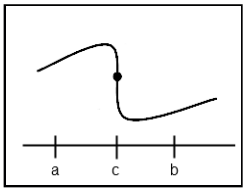


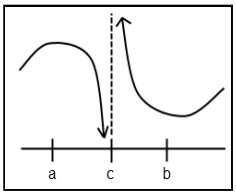


CASE III:

x	a	c	b
$f''(x)$	$-$	DNE	$+$
concavity	$\cap$	$*$	$\cup$

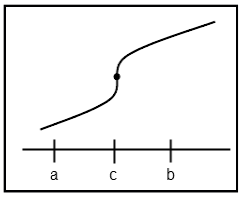


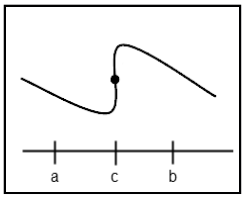


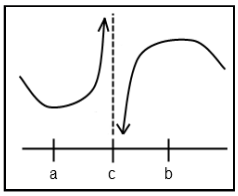


CASE IV:

x	a	c	b
$f''(x)$	$+$	DNE	$-$
concavity	$\cup$	$*$	$\cap$







References

[1] S. Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences*. Brooks Cole, Belmont, CA, 5th Edition, 2008.