SUMMARY OF LIMITS & CONTINUITY

NOTATIONS FOR LIMITS:

$\lim_{x\to a} f(x) = L$ means The limit of $f(x)$ as x approaches a is L.

$\lim_{x\to a^-} f(x) = L$ means The limit of $f(x)$ as x approaches a from the **left** is L.

$\lim_{x\to a^+} f(x) = L$ means The limit of $f(x)$ as x approaches a from the **right** is L.

$\lim_{x\to \infty} f(x) = L$ means The limit of $f(x)$ as x **increases without bound** is L.

$\lim_{x\to -\infty} f(x) = L$ means The limit of $f(x)$ as x **decreases without bound** is L.

RELATIONSHIP BETWEEN ONE-SIDED & TWO-SIDED LIMITS:

$\lim_{x\to a} f(x) = L \iff \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L.$

$\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x) \Rightarrow \lim_{x\to a} f(x)$ does not exist (DNE).

INDETERMINATE FORMS: $\frac{0}{0}, \frac{\infty}{\infty}$

If direct evaluation of a limit yields an indeterminate form, then try simplifying the function.
Consider factoring, rationalizing the numerator or denominator, or dividing by the the highest degree monic term.

LIMIT RULES:

Constant Rule: $\lim_{x\to a} k = k$ [k is a real number]

Constant Multiple Rule: $\lim_{x\to a}[cg(x)] = c\lim_{x\to a} g(x)$ [c is a real number]

Sum/Difference Rule: $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

Product Rule: $\lim_{x\to a}[f(x)g(x)] = \left[\lim_{x\to a} f(x)\right]\left[\lim_{x\to a} g(x)\right]$

Quotient Rule: $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ [NOTE: $\lim_{x\to a} g(x) \neq 0$]

Power Rule: $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$ [n is a real number]

Exponential Rule: $\lim_{x\to a} k^{f(x)} = k^{[\lim_{x\to a} f(x)]}$ [k is a positive real number]

CONTINUITY OF A FUNCTION:

A function f is continuous at point $x = a$ if $f(a)$ is defined, $\lim_{x\to a} f(x)$ is defined, and $\lim_{x\to a} f(x) = f(a)$.

Polynomials are continuous everywhere.
Rational functions $\left[\frac{P(x)}{Q(x)},\ where\ P(x), Q(x)\ are\ polynomials\right]$ are continuous everywhere $Q(x) \neq 0$.

References

[1] S. Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences*. Brooks Cole, Belmont, CA, 5th Edition, 2008.