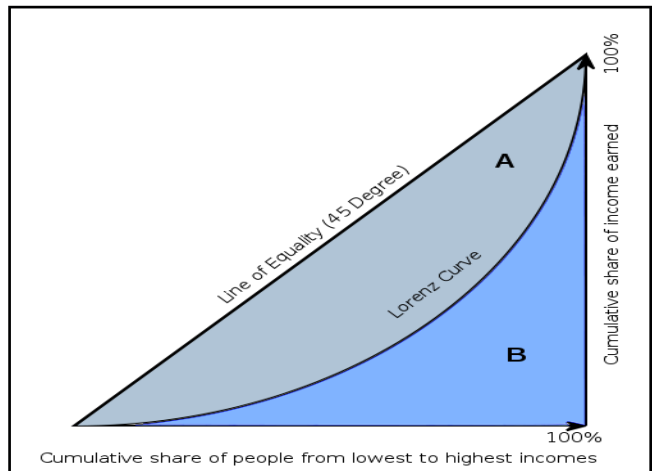


LORENZ CURVES

Lorenz curves measure **distribution of income** in a group, society or country.

A Lorenz curve $L(x)$ bears the following properties:

- The domain of L is $[0, 1]$.
- The range of L is $[0, 1]$.
- $L(0) = 0$ and $L(1) = 1$.
- $L(x) \leq x$ for every $x \in [0, 1]$.
- $L'(x) \geq 0$ on $[0, 1]$, meaning L increases on $[0, 1]$.



$L(x)$ denotes the **proportion** of the total income received by the **poorest** $100x\%$ of the population. For example, $L(0.45) = 0.25$ means that the poorest 45% of a group receive 25% of the total income.

The line $y = x$ is called the **Line of Equality**, meaning that income is equally distributed.

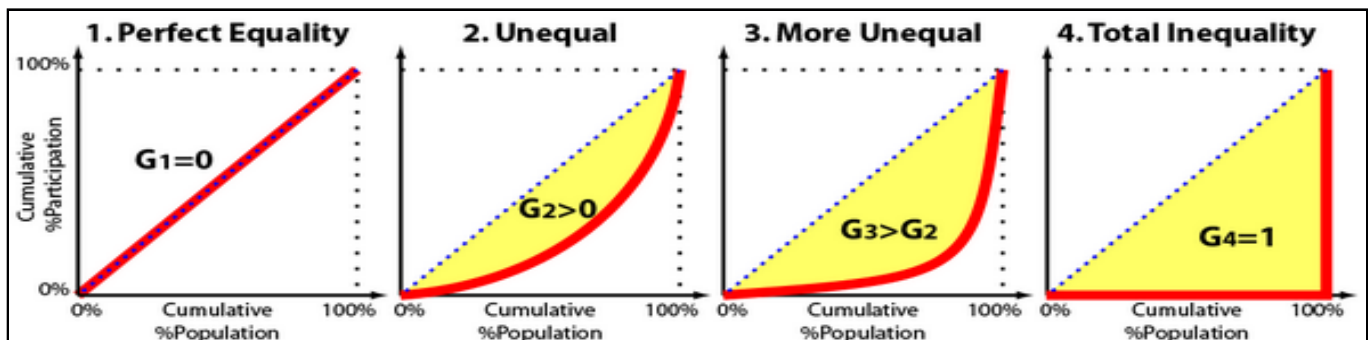
The **coefficient of inequality (Gini index)** of a Lorenz curve is : $G = 2 \int_0^1 [x - L(x)] dx$

Geometrically, the Gini index is the **ratio** of the area between the line $y = x$ and the Lorenz curve $L(x)$ to the area under the line $y = x$.

The lower the Gini index (G), the more equitable the income distribution.

If $G = 0$, which means the Lorenz curve is the line $L(x) = x$, then the group/society is called **egalitarian**.

If $G = 1$, which means the Lorenz curve is the line $L(x) = 0$, then the group/society is called **totalitarian**.



References

- [1] S. Tan, *Applied Mathematics for the Managerial, Life, and Social Sciences*. Brooks Cole, Belmont, CA, 5th Edition, 2008.
- [2] Lorenz Curve image from Wikipedia.
http://en.wikipedia.org/wiki/File:Economics.Gini_coefficient2.svg
- [3] Equality Comparison image from Dr. Michael Wu's blog on Lithosphere
<http://lithosphere.lithium.com>