Regression Chapter

Introduction

The textbook *Calculus: Applications and Technolgy* by Tan does include a section about linear regression equations, but Texas Tech Math Department and Business College wanted you to have more exposure to regression analysis. This chapter serves three purposes. The first purpose was to add to the material provided in the book, giving students a broader understanding of regression analysis, seeing it used for equqstions other that linear, and to see how one could decide upon which model is the best.

The second purpose of this chapter is to show how to use the calculator, as that is how nearly any instructor would have you calculate a regression equation or correlation coefficient. Since most instructors I know (including myself) are more familiar with the Texas Instrument (TI) calculators, that is the information I have provided. Screen shots and directions provided are for the TI-83+, which should be very similar to the TI-82, TI-83, and TI-84+ and any special editions of these calculators. Screen shots and directions are also provided for the TI-89 Titanium. In some cases, it may be similar to the TI-89, but in other cases in is drastically different. Since TI quit selling the 89 by 2004, I decided to not include it in this book. TI-86 is also another possible calculator, but from experience, I would say that less than 3% of students at Texas Tech have an 86, so I have omitted this at well. If you have a TI-89 or TI-86 and would like some help with the regession, please feel free email me (julia.head@ttu.edu) for a quick reference quide or to set up an appointment for help.

The third purpose of this chapter is to give examples of the applications of word problems in business. This will primarily be done by looking at regression equations of price, Cost, Revenue and Profit and maximizing or minimizing these equations or performing a profit-loss analysis. In emphasizing the use of the calculator, I will also show how to use your calculator to perform these actions, whereas we did these algebraically in the Function Review Chapter.

The bulk of the proofreading (so far) has been done by Lynn Carter, who I am very grateful and appreciative for all her work in this area. Thanks to Rachel Cline Backlund and Mandi Wheeler for their help during the writing and editing phases. Additional thanks to Dr. Robert Byerly his initial teaching and continued help with LaTeX. Thanks to Mandi Wheeler for allowing me to include some of her work in this chapter. As always, thanks to my father, Dr. James Head, for his support, encouragement, help, and inspiration in all my mathematical endeavors, including this one.

This is, however, still the second publication of this chapter and may contain errors. If you find any, please email julia.head@ttu.edu with the page number and error. Additionally, if you have any praise or constructive criticism about this chapter, you may email me. Thank you in advance for this help.

One final note: I have used 4 decimal places throughout most of this chapter (except when dealing with money or number of items), and expect students to do the same.

R-0   What is regression analysis? What is a correlation coefficient?

To understand what regression analysis is, we will begin by looking at two examples you may remember from Math 1330 (Barnett, Ziegler, and Byleen).

(Section 1-3 Example 7) At the beginning of the twenty-first century, the world demand for crude oil was about 75 million barrels per day and the price of a barrel fluctuated between \$20 and \$40. Suppose that the daily demand for crude oil is 76.1 million barrels when the price is \$25.52 per barrel and this demand drops to 74.9 million barrels when the price rises to \$33.68. Assuming a linear relationship between the demand x and the price p, find a linear function in the form $p = ax + b$ that models the price-demand for crude oil.

Since there are only two points, we find the slope between the points, (76.1, 25.52) and (74.9, 33.68):

$$
\begin{aligned}
m \quad &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{25.52 - 33.68}{76.1 - 74.9} \\
&= \frac{-8.16}{1.2} = -6.8
\end{aligned}
$$

Then using point-slope:

$$
\begin{aligned}
y \quad &= mx + b \\
25.52 \quad &= -6.8(76.1) + b \\
25.52 \quad &= -517.48 + b \\
543 \quad &= b
\end{aligned}
$$

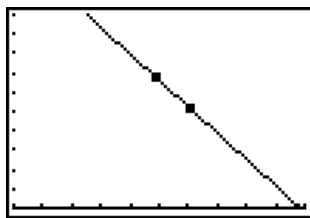So, $p = -6.8x + 543$.

Graph the points and the line.

Figure 1: Price-demand Graph

Does the line match the points exactly? Of course, because that's how we calculated it.

What if we had three points? Would they have to lie in a straight line? No, sometimes they will, but most of the time, once there are more than two points, we can't be exact anymore. But how do we determine the line that best fits the data?

(Section 1-1 Example 7) A manufacturer of a popular automatic camera wholesales the camera to retail outlets throughout the United States. Using statistical methods, the financial department in the company produced the price-demand data in Table 1, where p is the wholesale price per camera at which x million cameras are sold. Notice that as the price goes down, the number sold goes up.

x (Millions)	p (\$)
2	87
5	68
8	53
12	37

Table 1: Price-Demand

Using special analytical techniques (**regression analysis**), an analyst arrived at the following price-demand function that models the Table 1 data:

$$p(x) = 94.8 - 5x \qquad 1 \le x \le 15$$
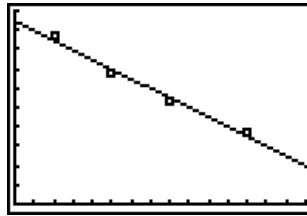
Part (A) asks you to plot the data and the line.

Figure 2: Price-demand Graph

Regression analysis is the process of fitting a function to a set of data points.

Does the line exactly match the data? No, no line will exactly match the data, but regression techniques tell us this is the best line that matches the data, so it's the one we will use.

To determine how well a line fits, we look at the **correlation coefficient** (R), a number between -1 and 1. If the correlation coefficient is 1, then the points all lie exactly in a line with a positive slope (See Figure 3). If the correlation coefficient is -1 then the points lie exactly in a line with negative slope (See Figure 4). If the correlation coefficient is 0 then there is no relationship between the points at all (See Figure 5).
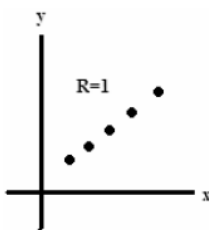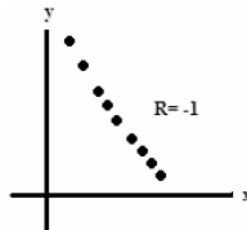

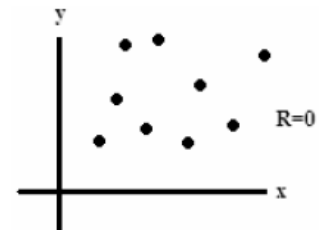Figure 3: Positive Correlation

Figure 4: Negative Correlation

Figure 5: No Correlation

We might also want to use data to find equations for things other than lines - like parabolas we have studied for revenue, profit, and cost. We can still use a correlation coefficient, but normally we are more interested in R^2. The higher this number, the better our data (points) fit the equation.

R-1   Calculating a regression equation

Well, it's great to understand the regression equations if someone gives them to you, but what if you were given the data and asked to find the equation? The answer is we will use our calculator to find the answer.

Example 1 (Sales Analysis). Merck & Co., Inc. is the world's largest pharmaceutical company. Their sales and income for the late 1990's are found in Table 1.

	1995	1996	1997	1998	1999
Sales	16.7	19.8	23.6	26.9	32.7
Income	3.3	3.8	4.6	5.2	5.9

Table 1: Selected Financial Data for Merck & Co., Inc. (Billion $)

(A) What linear equation, $S(x)$ best fits the sales data, where $x = 0$ corresponds to 1995?

(B) What is the correlation coefficient?

(C) Use the equation to estimate sales for Merck in 2005.

(D) Complete the table below.

x	Sales	$f(x)$
0	16.7	
1	19.8	
2	23.6	
3	26.9	
4	32.7	

Solution

Use your calculator and follow the steps for your calculator.

TI-83+

Step 1: Enter the list in the calculator

1. Press the STAT button on the calculator. This is the main statistics screen/menus. (See Figure 1)

2. When you have highlighted Edit... Press Enter or press the number corresponding it. (See Figure 2)

3. You should see L1 in the first column and L2 in the second column. Enter the x (years since 1995) into L1 (pressing enter after each). Scroll to the right and enter the Sales data into L2 so that each Sales on the corresponds to the correct x value on the left. (See Figure 3)

Step 2: Find the regression Equation

1. Press STAT again and this time scroll right until the top is highlighted CALC. Since the question asks for *linear* regression, scroll down to LinReg(ax+b) or press the number to which it corresponds. (See Figure 4)
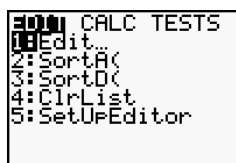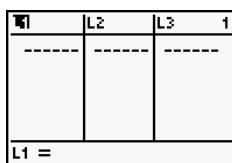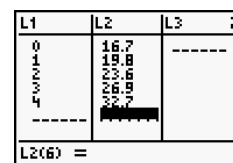
Figure 1: STAT Screen

Figure 2: List Screen

Figure 3: List Screen
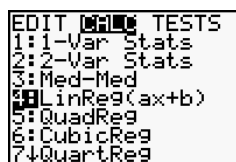

Figure 4: STAT CALC Screen
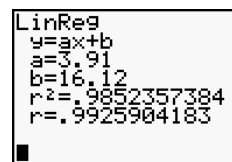

Figure 5: Regression Equation (Linear)

2. That should take you to the home screen where it displays LinReg(ax+b). Press Enter. You will see numbers for a and b. Since it says LinReg is ax+b, $S(x) = 3.91x + 16.12$. (See Figure 5)

TI-89 Titanium

Step 1: Enter the list in the calculator

1. Press the APPS button on the calculator and find the icon Stat/Lists Editor and press enter. (See Figure 6) If this is your first time using the Stats/List Editor, you may see the screen displayed in Figure 7. Press Enter.

2. Find list1 and list2. Enter the x (years since 1995) into list1 (pressing enter after each). Scroll to the right and enter the Sales data into list2 so that each Sales on the corresponds to the correct x value on the left. (See Figure 8)
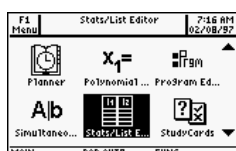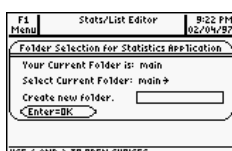

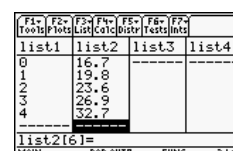Figure 6: APPS Screen

Figure 7: List Screen

Figure 8: List Screen

Step 2: Calculate the regression Equation

1. From the same screen, press F4 (Calculate) scroll to Regressions and press the right arrow. (See Figure 9)

2. Since the question asks for *linear* regression, scroll down to LinReg(ax+b) and press enter or press the number to which it corresponds. (See Figure 10)

3. That will bring up a screen similar to in Figure 11. Type in list1 for the X list and list2 for the Y list. Also make sure that it will store the RegEqn in y1(x). (We will use that later on.)

4. That will (a few seconds later) bring up a screen similar to the on shown in Figure 12. You will see numbers for a and b. Since it says LinReg is ax+b, $S(x) = 3.91x + 16.12$.
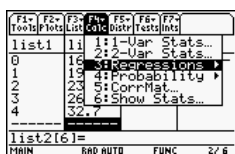
Figure 9: Calc Menu   Figure 10: Regression Menu   Figure 11: Information   Figure 12: Regression Equation

(A) So $S(x) = 3.91x + 16.12$

(B) The correlation coefficient is $R = .99259$.

(C) Use your calculator to find $S(10)$

TI-83+

Step 1: Put the Regression Equation into your calculator This one would be fairly easy to type in, but they are usually messier, so we will have the calculator put the equation in for us.

1. Press the Y= button. If you have anything entered for any of your Y's, clear them out. (See Figure 13)

2. With your cursor in Y1=, press the VARS button. Scroll down to or press the number corresponding to Statistics... (See Figure 14)

3. Scroll right to EQ (at the top) and then press enter on RegEq. (See Figure 15)
   Note that this only works since the last regression equation we calculated was the one we want to graph.

Your regression equation should appear in Y1 (See Figure 16)
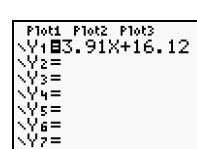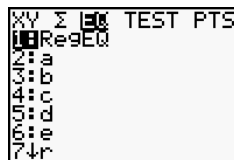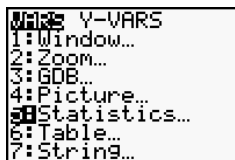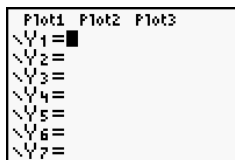
Figure 13: Y= Screen   Figure 14: VARS Screen   Figure 15: RegEq Option   Figure 16: Y=Regression

Step 2: Set the Window and Graph the regression equation

1. Press the WINDOW button.

2. Refer to your data to set your Xmin, Xmax, Ymin, and Ymax. Note that I chose Xmin=0 since that was the smallest x-value of my data. I chose Xmax=10 since in part (C) we are looking for $S(10)$ and that is larger that any of the x values the data. I chose Ymin=0 since that is slightly smaller the my lowest Sales data and Ymax=40 since it is slightly larger than the largest Sales data. (See Figure 17)

3. Press Graph (See Figure 18)

Figure 17: WINDOW Screen

Figure 18: Graph

Step 3: Use Trace to evaluate the regression equation for any value of x (between 0 and 10 since that's how we set the window)

1. Press the TRACE button and type in the value of x you want to evaluate for, in this case 10, and press enter. (See Figure 19)

2. The Sales data (y value) will appear in the lower right corner. (See Figure 20)
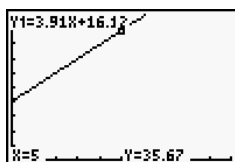
Figure 19: TRACE Screen
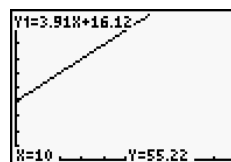
Figure 20: Evaluation

So $S(10) = 55.22$

TI-89 Titanium

Step 1: Set the Window and Graph the regression equation

1. Select WINDOW ($\Diamond$ F2).
   Refer to your data to set your xmin, xmax, ymin, and ymax. Note that I chose xmin=0 since that was the smallest x-value of my data. I chose xmax=10 since in part (C) we are looking for $S(10)$ and that is larger that any of the x values the data. I chose ymin=0 since that is slightly smaller the my lowest Sales data and ymax=40 since it is slightly larger than the largest Sales data. (See Figure 21)
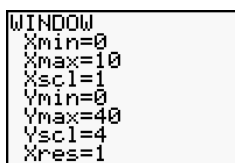
2. Press GRAPH ($\Diamond$ F3). (See Figure 22)

Figure 21: TRACE Screen
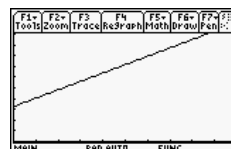
Figure 22: Evaluation

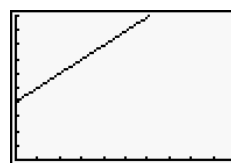**Step 2**: Use Trace to evaluate the regression equation for any value of x (between 0 and 10 since that's how we set the window)

(D) You can fill in the table by repeating the same process described in (C), using the trace feature.

x	Income	$f(x)$
0	16.7	16.92
1	19.8	20.03
2	23.6	23.94
3	26.9	27.85
4	32.7	31.76

Note that for both parts (C) and (D), you could have just used basic arithmetic, but as the regression equations get longer (with more decimals) this process will be more accurate.

Matched Problem 1 (Income Analysis)**.** Refer to the Financial Data from Example 1.

(A) What linear equation, $I(x)$ best fits the income data, where $x = 0$ corresponds to 1995?

(B) What is the correlation coefficient? What is R^2?

(C) Use the equation to estimate income for Merck in 2005.

(D) Complete the table below.

x	Sales	$f(x)$
0	3.3	
1	3.8	
2	4.6	
3	5.2	
4	5.9	

As we will see with the remaining examples, calculating any kind of regression equation is the same as the above process, but sometimes we will want a quadratic regression, cubic regression, exponential regression, etc. The only difference between the remaining examples and the first, is what type of regression we want.

Example 2 (Tire Mileage)**.** An automobile manufacturer collected the data in Table 2 relating tire pressure (x) in pounds per square inch and mileage in thousands of miles.

x	Mileage
28	45
30	52
32	55
34	51
36	47

Table 2: Tire Mileage

(A) What quadratic equation, $T(x)$ best fits the tire mileage data?

(B) What is R^2?

(C) Use the equation to estimate the mileage for a tire pressure of 31 pounds per square inch.

(D) Plot the points and graph the equation of the same axes.

(E) Complete the table below.

x	Mileage	$T(x)$
28	45	
30	52	
32	55	
34	51	
36	47	

Solution

TI-83+
Use the same steps as Example 1, but instead of using LinReg (option 4), use QuadReg (option 5).

TI-89 Titanium
Use the same steps as Example 1, but instead of using LinReg(ax+b) (option 2), use QuadReg (option 4).

For either calculator , you use the same process at Example 1 to plug the regression equation into your Y= menu and then evaluate the regression equation for different x values.

(A) So $T(x) = -.51758x^2 + 33.29286x - 480.94286$

(B) $R^2 = .95268$.

(C) $T(31) = 53.475$

(E) Table:

x	Mileage	$T(x)$
28	45	45.25714
30	52	51.77143
32	55	54.14286
34	51	52.37143
36	47	46.45714

Matched Problem 2 (Automobile Production)**.** Table 3 shows the retail market share of passenger cars from Ford Motor Company as a percentage of the U.S. Market.

Year	Market Share
1975	23.6%
1980	17.2%
1985	18.8%
1990	20.0%
1995	20.7%

Table 3: Ford Motor Company's Market Share

(A) What quadratic equation, $M(x)$ best fits the data, where $x = 0$ corresponds to 1975?

(B) What is R^2?

(C) Use the equation to estimate Ford's share of the market (as a percentage) in 2000.

(D) Complete the table below.

Year	Market Share	$M(x)$
1975	23.6%	
1980	17.2%	
1985	18.8%	
1990	20.0%	
1995	20.7%	

Example 3 (Fish Weights). Using the length of a fish to estimate its weight is of interest to both scientists and sports anglers. The data in Table 4 give the average weights of lake trout for certain lengths.

Length (in.)	Weight (oz.)	Length (in.)	Weight (oz.)
10	5	30	152
14	12	34	226
18	26	38	326
22	56	44	536
26	96		

Table 4: Lake Trout

(A) What cubic equation, $W(x)$ best fits the data, where x is the length of a length lake trout and $W(x)$ is the weight of the trout?

(B) What is R^2?

(C) Use the equation to estimate the weight if a lake trout that is 41 inches long.

(D) Complete the table below.

Length (in.) (x)	Weight (oz.)	$W(x)$
14	12	
22	56	
30	152	
38	326	
44	536	

Solution

TI-83+
Use the same steps as Example 1, but instead of using LinReg (option 4), use CubicReg (option 6).

TI-89 Titanium
Use the same steps as Example 1, but instead of using LinReg(ax+b) (option 2), use CubicReg (option 5).

For either calculator , you use the same process at Example 1 to plug the regression equation into your Y=
menu and then evaluate the regression equation for different x values.

(A) So $W(x) = .009517x^3 - .20682x^2 + 3.22761 - 17.54438$

(B) $R^2 = .99987$

(C) $W(41) = 423.02463$

(D) Table:

Length (in.)	Weight (oz.)	$W(x)$
14	12	13.21888
22	56	54.69537
30	152	150.09637
38	326	328.65747
44	536	534.73917

Matched Problem 3 (More Fish Weights). The data in Table 5 give the average weights of pike for certain
lengths.

Length (in.)	Weight (oz.)	Length (in.)	Weight (oz.)
10	5	30	108
14	12	34	154
18	26	38	210
22	44	44	326
26	72	52	522

Table 5: Pike

(A) What cubic equation, $W(x)$ best fits the data, where x is the length of a pike and $W(x)$ is the
weight of the pike?

(B) What is R^2?

(C) Use the equation to estimate the weight if a lake trout that is 47 inches long.

(D) Complete the table below.

Length (in.) (x)	Weight (oz.)	$W(x)$
14	12	
18	26	
26	72	
34	154	
44	326	

Example 4 (Depreciation)**.** Table 6 gives the market value of a particular model minivan (in dollars) x years after its purchase.

x	Value($)
1	12,575
2	9,455
3	8,115
4	6,845
5	5,225
6	4,485

Table 6: Minivan's Value

(A) What exponential equation, $V(x)$ best fits the data, where x is the number of years after purchase and $V(x)$ is value of the vehicle after x years?

(B) What is R^2?

(C) Use the equation to estimate the purchase price of the vehicle and the value of the vehicle after 8 years.

(D) Complete the table below.

x	Value($)	$V(x)$
1	12,575	
2	9,455	
3	8,115	
4	6,845	
5	5,225	
6	4,485	

Solution

TI-83+
Use the same steps as Example 1, but instead of using LinReg (option 4), use ExpReg (option 9).

TI-89 Titanium
Use the same steps as Example 1, but instead of using LinReg(ax+b) (option 2), use ExpReg (option 0).

For either calculator , you use the same process at Example 1 to plug the regression equation into your Y= menu and then evaluate the regression equation for different x values.

(A) So $V(x) = 14910.2031 \cdot .8163^x$

(B) $R^2 = .9917$.

(C) $V(0) = 14,910.20$ and $V(8) = 2939.39$

(E) Table:

x	Value($)	$V(x)$
1	12,575	12,171.11
2	9,455	9,935.20
3	8,115	8,110.05
4	6,845	6,620.18
5	5,225	5,404.02
6	4,485	4,411.27

Matched Problem 4 (Depreciation). Table 7 gives the market value of a luxury sedan (in dollars) x years after its purchase.

x	Value($)
1	23,125
2	19,050
3	15,625
4	11,875
5	9,450
6	7,125

Table 7: Luxury Sedan's Value

(A) What exponential equation, $V(x)$ best fits the data, where x is the number of years after purchase and $V(x)$ is value of the vehicle after x years?

(B) What is R^2?

(C) Use the equation to estimate the purchase price of the vehicle and the value of the vehicle after 10 years.

(D) Complete the table below.

x	Value($)	$V(x)$
1	23,125	
2	19,050	
3	15,625	
4	11,875	
5	9,450	
6	7,125	

Example 5 (Supply and Demand). A cordless screwdriver is sold through a national chain of discount stores. A marketing company established price-supply and price-demand in Table 8, where x is the number of screwdrivers in a month, $S(x)$ is the price-supply and $D(x)$ is the price-demand.

x	$S(x)$	$D(x)$
1,000	9	91
2,000	26	73
3,000	34	64
4,000	38	56
5,000	41	53

Table 8: Price-Supply and Price-Demand for Screwdrivers

(A) What logarithmic equation, $S(x)$ best fits the supply data?

(B) What is R^2?

(C) Estimate the price per screwdriver at a supply of 1500 screwdrivers.

(D) Complete the table below.

x	Supply($)	$S(x)$
1,000	9	
2,000	26	
3,000	34	
4,000	38	
5,000	41	

Solution

TI-83+
Use the same steps as Example 1, but instead of using LinReg (option 4), use LnReg (option 9).

TI-89 Titanium
Use the same steps as Example 1, but instead of using LinReg(ax+b) (option 2), use LnReg (option 7).

Note that LnReg stands for Natural Log (ln), where LinReg stands for Linear.

For either calculator , you use the same process at Example 1 to plug the regression equation into your Y= menu and then evaluate the regression equation for different x values.

(A) So $S(x) = -127.8085 + 20.0131 \ln(x)$

(B) $R^2 = .9845$.

(C) $S(1500) = 18.55$

(E) Table:

x	Supply($)	$S(x)$
1,000	9	10.44
2,000	26	24.31
3,000	34	32.42
4,000	38	38.18
5,000	41	42.45

Matched Problem 5 (Supply and Demand)**.** Refer to the Demand data in Table 8.

(A) What logarithmic equation, $D(x)$ best fits the demand data?

(B) What is R^2?

(C) Estimate the price per screwdriver at a demand of 1800 screwdrivers.

(D) Complete the table below.

x	Demand($)	$D(x)$
1,000	91	
2,000	73	
3,000	64	
4,000	56	
5,000	53	

There are many other types of regression models: quartic $(ax^4 + bx^3 + cx^2 + dx + k)$, Power $(a \cdot x^b)$, and Logistic $\left(\dfrac{c}{1 + ae^{-bx}} \right)$ to name a few. They all work similarly to the ones mentioned already.

ANSWERS TO MATCHED PROBLEMS

1. (A) $I(x) = .66x + 3.24$ (B) $R = .99817$ $R^2 = .99634$ (C) $I(10) = 9.84$ (D)

x	Sales	$f(x)$
0	3.3	3.24
1	3.8	3.90
2	4.6	4.56
3	5.2	5.22
4	5.9	5.88

2. (A) $M(x) = .03949x^2 - .84857x + 22.6314$ (B) $R^2 = .63855$ (C) $M(25) = 26.06$

(D)

Year	Market Share	$M(x)$
1975	23.6%	22.6314
1980	17.2%	19.3743
1985	18.8%	18.0886
1990	20.0%	18.7743
1995	20.7%	23.4314

3. (A) $W(x) = .003111x^3 + .04057x^2 - .5341x + 3.3416$ (B) $R^2 = .9999$ (C) $W(47) = 390.8343$

(D)

Length (in.) (x)	Weight (oz.)	$W(x)$
14	12	12.3522
18	26	25.0150
26	72	71.5564
34	154	154.3493
44	326	323.3781

4. (A) $V(x) = 30363.1764(.7897)^x$ (B) $R^2 = .9953$ (C) $V(10) = 2863.53$ (D)

x	Value($)	$V(x)$
1	23,125	23,977.43
2	19,050	18,934.69
3	15,625	14,952.49
4	11,875	11,807.80
5	9,450	9,324.47
6	7,125	7,363.42

5. (A) $D(x) = 256.4659 - 24.0381 \ln(x)$ (B) $R^2 = .9960$ (C) D(1800)= 76.29 (D)

x	Demand($)	$D(x)$
1,000	91	90.42
2,000	73	73.75
3,000	64	64.01
4,000	56	57.09
5,000	53	51.73

R-2 Using Regression Equations

It is great to find a regression equation, but besides finding one in R-1, the only thing else that was asked of you in regards using that equation, was to evaluate it. Suppose the equation you found modeled a company's profit; then you should be able to use that equation to estimate the company's maximum profit.

Example 1 (Memory Cards). Klein & Cline Electronics gathered the following data summarized in Table 1 when selling x thousand memory cards at different prices, p.

x	p
38.1	$20
35.4	$25
32.2	$30
28.9	$35
25.0	$40

Table 1: Memory Cards Sales

(A) Find a linear regression model that will approximate the price, $p(x)$, as a function of the quantity demanded, x.

(B) Find a model, $R(x)$ that represents the company's revenue from memory cards.

(C) About how many memory cards should the company sell to maximize the revenue from memory cards? What is that revenue?

(D) Approximately how much should Klein & Cline sell the memory cards for to maximize the revenue?

Solution

(A) Using LinReg on the graphing calculator, we get $p(x) = -1.5225x + 78.5987$

(B) $R(x) = x \cdot p(x) = x(-1.5225x + 78.5987)$

(C) We could maximize with algebraic techniques we used in the first chapter; however, instead, since we already have the function in the calculator, we will look at using the calculator to help us. First, as we did in the previous section, enter the regression function in y1 (if you have an 89, you have already done this). However, we are not interested in maximizing the regression function, price, but revenue. So first, we need to multiply the function in y1 by x.

To set the window, the same techniques for setting the xmin and xmax will work, but we will need to use the table to set our ymin and ymax. As before, we should set our xmin at 20, since the lowest x value is 24. We would set the xmax around 40 since the highest x value is 38.1. We will do something similar for y's except instead of looking at the table of data, we will look at the table in the calculator.

TI-83+ This is found on the TI-83+, by going first to 2nd WINDOW (TBLESET). Set the TbleStart=20 (or whatever you chose for xmin) and ΔTbl as your step size. In this case, we are counting from 20 to 40, so it is reasonable to count by 1's, so I would set ΔTbl=1. The highlights shoukd be set at Auto and should not be changed. (See Figure 1.) To access your table, press 2nd Graph. (See Figure 2.)

Figure 1: Setting Table

Figure 2: Table

TI-89 Titanium The process is similar on the the TI-83+. In this case, TBLSET is found by using ◊F2. Set tblStart and Δtbl as described in the 83+. The other settings should be left alone. (See Figure 3.) You will need to hit enter twice. Then access the table by pressing ◊F5. (See Figure 4.)

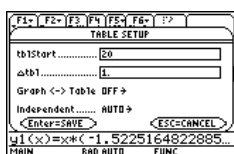Figure 3: Setting Table

Figure 4: Table

Looking at the table, look for the lowest and highest y value listed for y1 between 20 and 40 (or whatever you selected for xmin and xmax). Set your ymin below the lowest y value and your ymax above the highest y value. In this case, I chose to set my ymin at 700 and my ymax at 1000.

To find the maximum: With your window set, graph the function.

TI-83+ Press 2nd TRACE (CALC) and press 4:maximum (note this is where you will find minimum, zero, and intersect, which may also be helpful). When it asks for a Left Bound, arrow to the left of the maximum and press enter. (See Figure 5.) Similarly, when it asks for Right Bound, arrow to the right of the maximum and press enter. (See Figure 6.) It will then ask for a guess. Your cursor must be in between the left and right bounds; then hit enter. (See Figure 7.) It will then display the x and y coordinates for the maximum at the bottom. (See Figure 8.)
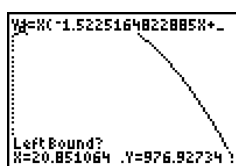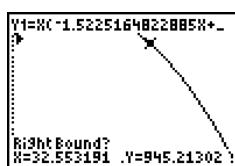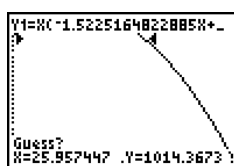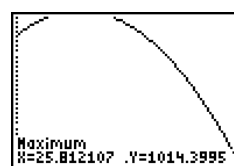

Figure 5: Left Bound

Figure 6: Right Bound

Figure 7: Guess

Figure 8: Maximum

TI-89 Titanium Press F5 (math graph menu) and select 4:Maximum (note this is where you will find minimum, zero, and intersection, which may also be helpful). When it asks for a Lower Bound, arrow to the left of the maximum and press enter. (See Figure 9.) Similarly, when it asks for Upper Bound, arrow to the right of the maximum and press enter. (See Figure refF:89TR3ex1f4.) It will then display the x and y coordinates for the maximum at the bottom. (See Figure 11.)
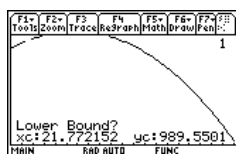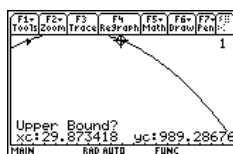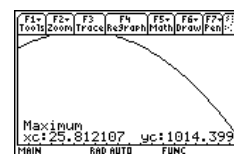
Figure 9: Lower Bound

Figure 10: Upper Bound

Figure 11: Maximum

Since this question asks how many memory cards should be produced and maximum revenue, we are looking for both the x value, so the answer is producing 25.8121 thousand memory cards will yield the maximum revenue of $1014.3995 thousand dollars or $1,104,399.50

(D) This question asks for the price that maximizes revenue. In part (C) we learned that they would need $x = 25.8121$ to maximize revenue, so we plug in this number into the price function.

$$p(25.8121) = -1.5225(25.8121) + 78.5987 = \$39.30$$

Note that when I plugged this is, even though my work shows the rounded numbers of the regression equations, I used the actual numbers (with all of the decimal places), so your answer may vary slightly.

Matched Problem 1 (Monitors). Osoinach Computers gathered data concerning price, p, and quantity demanded in hundreds of flatscreen monitors, x, summarized in Table 2 below.

x	p
76.3	$500
71.5	$600
68.9	$650
64.0	$750
61.3	$800

Table 2: Flatscreen Monitors

(A) Find a linear regression model that will approximate the price, $p(x)$, as a function of the quantity demanded, x.

(B) Find a model, $R(x)$ in hundreds that represents the company's revenue from flatscreen monitors.

(C) About how many memory cards should the company sell to maximize the revenue from flatscreen monitors? What is that revenue?

Example 2 (Profit-Loss Analysis). The Society for Industrial and Applied Mathematics (SIAM) has the following information in Table 3 about the costs, C in hundreds dollars, of selling x hundred T-shirts for an upcoming fundraiser:

x	C
3.2	$97.00
4.1	$83.20
5.0	$71.50
6.4	$71.00
7.9	$81.50
8.7	$95.50

Table 3: T-Shirt Costs

(A) Find the quadratic regression model, $C(x)$ in hundreds of dollars, that best approximates the organizations costs for T-Shirt.

(B) If SIAM sells the shorts for $20 each, find the model, $P(x)$ (in hundreds of dollars) that represents their Profit.

(C) How many shirts (at $20 each) does SIAM need to sell to maximize profit? What is the maximum profit?

(D) According to the model, how many shirts must SIAM sell to break even? To make a profit? To have a loss?

Solution

(A) Using QuadReg, $C(x) = 3.552x^2 - 42.6304x + 197.2737$

(B) To find profit, in addition to cost, you need the company's revenue function. $R(x) = x \cdot p = 20x$. So,
$$P(x) = R(x) - C(x) = 20x - (3.552x^2 - 42.6304x + 197.2737)$$

(C) As before, we can use the graphing calculator to find the maximum. To set our window this time, we see xmin=0 and xmax=10 would be acceptable choices. Using the table to find the y's, we can let ymax=90 and ymin=-150. By using the maximum, we get $x = 8.8161$ or 882 T-Shirts for a maximum profit of $78.8051 hundred dollars or $7880.51.

(D) We can also use the graphing calculator to find the break-even points. However, with the window we have set from (C), we only see one point, so we will need to increase the xmax. Since we can see one break-even point and the maximum, normally we can just double the current xmax to see the other break even-point. Setting xmax=20 (and leaving the other window settings the same as in (C)), you should see both break-even points. To find the break-even points, we are looking for the zeros of the profit function.

TI-83+ Use the math graph menu, 2nd TRACE (CALC), and this time select 2:Zero. We will have to go through this process twice: once for each break-even point. It does not matter which one you find first; I will usually find the smaller break-even point first (the one furthest left). So when it asks for Left Bound (See Figure 12.), I need to go to the left of the break-even point I am trying to find. When it asks for Right Bound, go to the right of the break-even point. (See Figure 13.) When you set these bounds, make sure that only one break-even point is between the bounds. Then go to that point and press enter. (See Figure 14.) It will give the answer at the bottom of the screen. (See Figure 15.) Repeat this process for the second break-even point. (See Figures 16 - 19.)
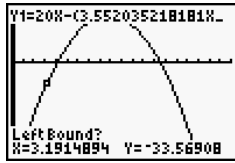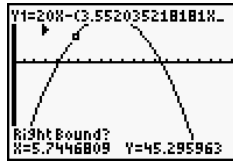
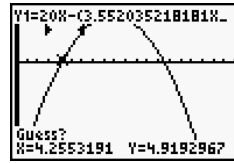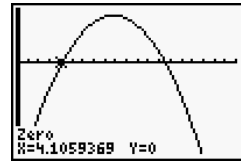Figure 12: Left Bound
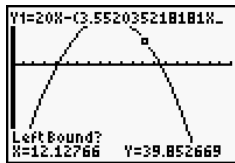

Figure 13: Right Bound

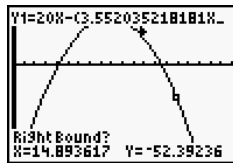Figure 14: Guess

Figure 15: Zero
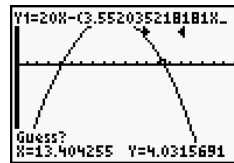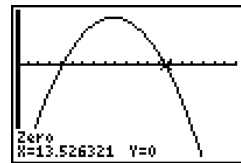

Figure 16: Left Bound

Figure 17: Right Bound

Figure 18: Guess

Figure 19: Zero

TI-89 Titanium Use the math graph menu, F5, and this time select 2:Zero. We will have to go through this process twice: once for each break-even point. It does not matter which one you find first; I will usually find the smaller break-even point first (the one furthest left). So when it asks for Lower Bound (See Figure 20.), I need to go to the left of the break-even point I am trying to find. When it asks for Upper Bound, go to the right of the break-even point. (See Figure 22.) When you set these bounds, make sure that only one break-even point is between the bounds. It will give the answer at the bottom of the screen. (See Figure 24.) When you repeat this process for the second break-even point, remember that Lower Bound is in terms of x, so in this case the Lower Bound is "higher" (with respect to the y-axis) that the Upper Bound. (See Figures 21, 23, and 25.)
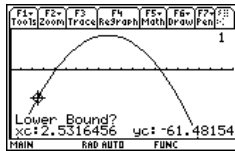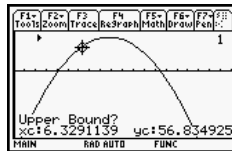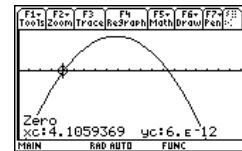

Figure 20: Lower Bound

Figure 22: Upper Bound
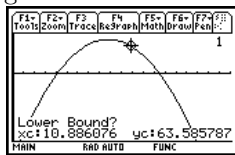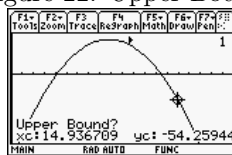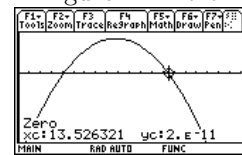

Figure 24: Zero

Figure 21: Lower Bound

Figure 23: Upper Bound

Figure 25: Zero

This gives you break-even points of 4.11 and 13.53 (or 411 and 1353 T-Shirts). You will make a profit when $4.11 < x < 13.53$ and have a loss when $0 \leq x < 4.11$ or $x > 13.53$.

Matched Problem 2 (Profit-Loss Analysis). The National Education Association (NEA) has the following information in Table 4 about the costs, C in thousands, of manufacturing x thousand professional journal subscriptions per year:

x	C
10.9	$3710
12.1	$2360
13.4	$1200
14.7	$1150
15.8	$2490
17.0	$3680

Table 4: Journal Costs

(A) Find the quadratic regression model, $C(x)$ that best approximates the associations costs for journals.

(B) If NEA sells the subscriptions for $150 each, find the model, $P(x)$ that represents their Profit.

(C) How many subscriptions (at $150 each) does NEA need to sell to maximize profit? What is the maximum profit?

(D) According to the model, how many subscriptions must NEA sell to break even? To make a profit? To have a loss?

ANSWERS TO MATCHED PROBLEMS
1. (A) $p(x) = 2030.0225 - 20.0296x$ (B) $R(x) = xp(x) = x(2030.0225 - 20.0296x)$ (C) maximize revenue when they sell 5068 monitors; maximum revenue is $5,143,634.70
2. (A) $C(x) = 274.4473x^2 - 7655.7205 + 54617.105$ (B) $P(x) = 150x - (274.4473x^2 - 7655.7205 + 54617.105)$ (C) maximum profit of $884,684.67 when selling 14,221 subscriptions (D) they break-even when $x = 12.245$ or when $x = 16.016$; profit when $12.245 < x < 16.016$; loss when $10.9 \leq x < 12.245$ or $16.016 < x \leq 17.0$

Exercise R-2

1. The following table relates the age of a female moose (in years) to offspring mortality during hunting season.

Age	2	3	4	5	6	7	8	9	10
Mortality	0.5	0.4	0.25	0.35	0.35	0.5	0.37	0.35	0.48

(A) Find the best fitting quadratic relating age to mortality rate.

(B) Find the age at which mortality of offspring is minimized. (Round to the nearest year)

2. The following table relates the age of a female antelope (in years) to offspring mortality during hunting season.

Age	3	4	5	6	7	8	9	10	11
Mortality	0.5	0.45	0.3	0.35	0.25	0.2	0.37	0.45	0.55

(A) Find the best fitting quadratic relating age to mortality rate.

(B) Find the age at which mortality of offspring is minimized. (Round to the nearest year)

3. The following table relates the number of items, x produced in millions, to the company's costs to produce x items in millions.

x	3	4	5	6	7	8	9	10	11
$C(x)$	0.5	0.4	0.25	0.35	0.35	0.5	0.37	0.48	0.55

(A) Find the best fitting quadratic model relating x to $C(x)$.

(B) Find the production level at which cost is minimized. (Round to the nearest item.)

4. The following table relates the number of items, x produced in millions, to the company's costs to produce x items in millions.

Age	2	3	4	5	6	7	8	9	10
Mortality	0.6	0.53	0.48	0.35	0.37	0.35	0.4	0.5	0.55

(A) Find the best fitting quadratic model relating x to $C(x)$.

(B) Find the production level at which cost is minimized. (Round to the nearest item.)

R-3   Determining the best model

In all of the previous examples and problems, you were told what model to use. But normally your company will give you data and you will need to determine which model to use.

The model that has the best R^2 value is the best; but you don't want to have to try every single one. Plot the points and see the general shape and then try the model(s) that resemble that pattern and take the one with the best R^2 value. However, balance the choice of R^2 with practicality. For example, if the R^2 for the cubic is only slightly better than the R^2 of the linear or quadratic, then take the less complicated linear or quadratic.

Example 1 (Comparing R^2 Values). Regression analysts at Johnson, Inc. found the following two equations and R^2 values that might model the company's costs. Which should they choose and why?

$$C(x) = .673x + 5.489 \quad R^2 = .9820$$
$$C(x) = .742x^2 + .258x + 2.580 \quad R^2 = .9745$$

Solution

Since the first equation is linear, which is simpler than a quadratic like the second equation, and the first equation has a higher R^2 value, you should choose the linear equation.

Matched Problem 1 (Comparing R^2 Values). The regression analysts at Johnson, Inc. also found the following two equations and R^2 values that might model the price of the company's top selling item. Which should they choose and why?

$$p(x) = .005x + .2566 \quad R^2 = .8515$$
$$p(x) = .250x^2 + 7.5x + 8.29 \quad R^2 = .9804$$

Example 2 (Depreciation). Xerox has found the depreciation of their premier copiers after x years to be as described in Table 1.

   (A) Calculate the linear, quadratic, cubic, and exponential regression models and the R^2 value for each.
   (B) Which has the higher R^2 value? Are there any R^2 values that are so low that those equations should not be considered?
   (C) Which equation should you probably choose? Why?

x years	Current Value
0	$2000
1	$1890
2	$1675
3	$1480
4	$1270
5	$1055

Table 1: Value of Xerox Premier Copiers

Solution

(A) Using the appropriate equation, you should get the following regressions equations and R^2 values. (See Figures 1 - 4 for the TI-83+ and Figures 5 - 8 for the TI-89 Titanium.)
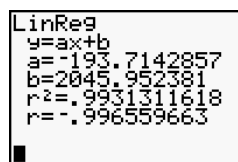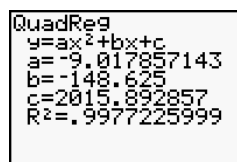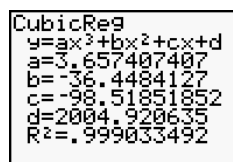

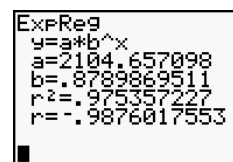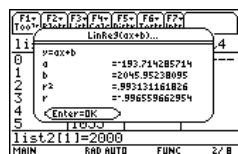Figure 1: Left Bound   Figure 2: Right Bound   Figure 3: Guess   Figure 4: Zero
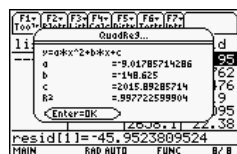

Figure 5: Left Bound   Figure 6: Right Bound   Figure 7: Guess   Figure 8: Zero

Linear	$V(x) = -193.7143x + 2045.9524$	$R^2 = .9931$
Quadratic	$V(x) = -9.0179x^2 - 148.625x + 2015.8929$	$R^2 = .9977$
Cubic	$V(x) = 3.6574x^3 - 36.4484x^2 - 98.5185x + 2004.9206$	$R^2 = .9990$
Exponential	$V(x) = 2104.6571 \cdot .8790^x$	$R^2 = .9753$

(B) Linear, Quadratic, and Cubic all have very close R^2 values, but Cubic has the highest. Since Exponential has a smaller R^2 value, you would probably not use it, although the R^2 is high enough that you would not discount it as an option.

(C) You could argue that cubic has the highest R^2 value, so choose that. You could also argue that Linear is close to cubic and linear is easier to use. In this case, there is no clear-cut answer, but whatever you choose, justify.

Matched Problem 2 (Costs). Xerox gathered the data summarized in Table 2 about the cost, $C(x)$ in thousands to produce x thousand premier copiers.

(A) Calculate the linear, quadratic, cubic, and logarithmic regression models and the R^2 value for each.

(B) Which has the higher R^2 value? Are there any R^2 values that are so low that those equations should not be considered?

x years	$C(x)$
100	$11,500
200	$9,200
300	$8,600
400	$9,700
500	$11,800
600	$14,300

Table 2: Cost of Xerox Premier Copiers

(C) Which equation should you probably choose? Why?

ANSWERS TO MATCHED PROBLEMS
1. The quadratic equation has a significantly higher R^2 value, so you should choose it.
2. (A) Linear: $C(x) = 6.5429x + 8560$ $R^2 = .3354$ Quadratic: $C(x) = .0621x^2 - 36.9571x + 14360$ $R^2 = .9809$ Cubic: $C(x) = -.00008x^3 + .1458x^2 - 62.1997x + 16366.6667$ $R^2 = .9993$ Logarithmic: $C(x) = 4355.5599 + 1130.0333 \ln x$ $R^2 = .3571$ (B) highest is the cubic; linear and logarithmic are too low (C) quadratic since its R^2 is very close to the cubic and quadratic is easier to use

Exercise R-3

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1. If you were given the following regression data for a company's costs, which model should you choose to fit the data the best and why?

   Linear   Quadratic   Cubic
   $C(x) = 1.554x - 58.269$ $C(x) = .001x^2 + 1.0381x + 2.7502$ $C(x) = .00003x^3 + .026025x^2 - 4.986x + 454.95$
   $R^2 = 9681$ $R^2 = 9698$ $R^2 = .982$

2. If you were given the following regression data for a computer's price, which model should you choose to fit the data the best and why?

   Linear   Quadratic   Cubic
   $C(x) = .623x + 44.72$ $C(x) = .00006x^2 + .668x + 37.81$ $C(x) = 6.37x^3 - .00675x^2 - 2.89x - 193.22$
   $R^2 = 9681$ $R^2 = 9682$ $R^2 = .974$

3. The data for a statistical estimation of the cost-output relationship for a firm is given below. Here x is the output in millions of units, and $C(x)$ is the total cost in thousands of dollars. Determine the best model to describe this data. Why did you choose that model?

x	50	60	73	82	90	215	230	260
$C(x)$	180	160	150	125	95	60	50	54

4. The data for a statistical estimation of the cost-output relationship for a firm is given below. Here x is the output in millions of units, and $C(x)$ is the total cost in thousands of dollars. Determine the best model to describe this data. Why did you choose that model?

x	5	6	11	16	22	26	29	32
$C(x)$	680	720	760	950	1450	1650	1450	1650

5. The data for a statistical estimation of the cost-output relationship for a firm is given below. Here x is the output in millions of units, and $C(x)$ is the total cost in thousands of dollars. Determine the best model to describe this data. Why did you choose that model?

x	50	65	75	80	85	90	92	97
$C(x)$	18	22	15	13	19	23	22	25

6. The data for a statistical estimation of the cost-output relationship for a firm is given below. Here x is the output in millions of units, and $C(x)$ is the total cost in thousands of dollars. Determine the best model to describe this data. Why did you choose that model?

x	50	60	110	160	220	260	290	320
$C(x)$	680	720	760	950	1450	1650	1450	1650