CURVE SKETCHING (II): CUSPS, VERTICAL TANGENTS, AND ASYMPTOTES

Throughout this page, assume $y = f(x)$ and f is an **elementary function** (not piecewise and not implicit).

ASYMPTOTES: $\left[k, c_1, c_2 \in \mathbb{R}\right]$

- $y = k$ is a **horizontal asymptote** of $f \iff \left[\lim_{x\to\infty} f(x) = k \quad \text{OR} \quad \lim_{x\to-\infty} f(x) = k\right]$
- $x = k$ is a **vertical asymptote** of $f \iff \left[\lim_{x\to k^+} f(x) = \pm\infty \quad \text{OR} \quad \lim_{x\to k^-} f(x) = \pm\infty\right]$
- $y = c_1x + c_2$ is a **slant asymptote** of $f \iff \left[\lim_{x\to\infty} [f(x) - (c_1x + c_2)] = 0 \quad \text{OR} \quad \lim_{x\to-\infty} [f(x) - (c_1x + c_2)] = 0\right]$
- A function can have **zero, one, or two horizontal or slant asymptotes**.
- A fucntion can have **infinitely many vertical asymptotes** (e.g. $f(x) = \tan x$)
- A function **never touches or crosses a vertical asymptote**, but this is not necessarily true for horizontal asymptotes.

HILLS & VALLEYS: $\left[-\infty < x_L < c < x_R < \infty\right]$

- f has a **hill** at $x = c \iff$ the **slope table near** $x = c$ looks like

x	$\cdots$	x_L	c	x_R	$\cdots$
$f'(x)$	$\cdots$	$+$	0	$-$	$\cdots$
slope	$\cdots$	/	–	\	$\cdots$

- f has a **valley** at $x = c \iff$ the **slope table near** $x = c$ looks like

x	$\cdots$	x_L	c	x_R	$\cdots$
$f'(x)$	$\cdots$	$-$	0	$+$	$\cdots$
slope	$\cdots$	\	–	/	$\cdots$

CUSPS:

- f has a **cusp** at $x = c \iff \left[\lim_{x\to c^-} f'(x) = -\infty \text{ AND } \lim_{x\to c^+} f'(x) = +\infty\right]$ OR $\left[\lim_{x\to c^-} f'(x) = +\infty \text{ AND } \lim_{x\to c^+} f'(x) = -\infty\right]$
- f has an **up-cusp** at $x = c \iff$ the tables **near** $x = c$ are

x	x_L	c	x_R
$f'(x)$	$+$	DNE	$-$
slope	/	|	\

AND

x	x_L	c	x_R
$f''(x)$	$+$	DNE	$+$
concavity	$\cup$	$*$	$\cup$

- f has a **down-cusp** at $x = c \iff$ the tables **near** $x = c$ are

x	x_L	c	x_R
$f'(x)$	$-$	DNE	$+$
slope	\	|	/

AND

x	x_L	c	x_R
$f''(x)$	$-$	DNE	$-$
concavity	$\cap$	$*$	$\cap$

VERTICAL TANGENTS: $\left[-\infty < x_L < c < x_R < \infty\right]$

- f has a **vertical tangent** at $x = c \iff \left[\lim_{x\to c^-} f'(x) = \lim_{x\to c^+} f'(x) = +\infty\right]$ OR $\left[\lim_{x\to c^-} f'(x) = \lim_{x\to c^+} f'(x) = -\infty\right]$
- Occurs at $x = c \iff$ the tables **near** $x = c$ are

x	x_L	c	x_R
$f'(x)$	$-$	DNE	$-$
slope	\	|	\

AND

x	x_L	c	x_R
$f''(x)$	$-$	DNE	$+$
concavity	$\cap$	$*$	$\cup$

- Occurs at $x = c \iff$ the tables **near** $x = c$ are

x	x_L	c	x_R
$f'(x)$	$+$	DNE	$+$
slope	/	|	/

AND

x	x_L	c	x_R
$f''(x)$	$+$	DNE	$-$
concavity	$\cup$	$*$	$\cap$

- **Vertical tangents** are also **inflection points**.

RELATIVE EXTREMA:

- **Relative (local) maxima** of f occur at either **hills**, **up-cusps**, or **corners**.
- **Relative (local) minima** of f occur at either **valleys**, **down-cusps**, or **corners**.

EXAMPLE: Let $f(x) = \dfrac{x(x-1)(x-2)^2(x-3)}{(x-1)(x-2)^2(x-3)^3(x+4)^{5/3}\sqrt[5]{x+5}}$. Find all horizontal & vertical asymptotes.

Since f is a **rational function**, notice f is undefined when the **denominator** is zero which is when $x \in \{1, 2, 3, -4, -5\}$

Now, use the definition to determine if a **vertical asymptote** really occurs at each of these **suspected** x-values:

$$\lim_{x\to 0^-} f(x) \overset{NS}{=} \frac{(0)(-1)(4)(-3)}{(-1)(4)(-27)(4)^{5/3}(1)} = 0 \qquad \lim_{x\to 0^+} f(x) \overset{NS}{=} \frac{(0)(-1)(4)(-3)}{(-1)(4)(-27)(4)^{5/3}(1)} = 0$$

Since $\lim\limits_{x\to 0^-} f(x) \neq \pm\infty$ and $\lim\limits_{x\to 0^+} f(x) \neq \pm\infty$, $x = 0$ is NOT a vertical asymptote.

$\lim\limits_{x\to 1} f(x) \overset{NS}{=} \dfrac{0}{0} \implies$ Rewrite/simplify function $\implies$ cancel the $(x-1)$ terms on top & bottom of $f(x)$

Hence, $\lim\limits_{x\to 1} \dfrac{x(x-2)^2(x-3)}{(x-2)^2(x-3)^3(x+4)^{5/3}\sqrt[5]{x+5}} \overset{NS}{=} \dfrac{(1)(1)(-2)}{(1)(-8)(5)^{5/3}\sqrt[5]{6}} \neq \pm\infty$

Since $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) \neq \pm\infty$, $x = 1$ is NOT a vertical asymptote.

$\lim\limits_{x\to 2} f(x) \overset{NS}{=} \dfrac{0}{0} \implies$ Rewrite/simplify function $\implies$ cancel the $(x-2)^2$ terms on top & bottom of $f(x)$

Hence, $\lim\limits_{x\to 2} \dfrac{x(x-1)(x-3)}{(x-1)(x-3)^3(x+4)^{5/3}\sqrt[5]{x+5}} \overset{NS}{=} \dfrac{(2)(1)(-1)}{(1)(-1)(6)^{5/3}\sqrt[5]{7}} \neq \pm\infty$

Since $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) \neq \pm\infty$, $x = 2$ is NOT a vertical asymptote.

$$\lim_{x\to 3^-} f(x) \overset{NS}{=} \frac{6}{(2)(7)^{5/3}\sqrt[5]{8}}\left[\lim_{x\to 3^-}\frac{x-3}{(x-3)^3}\right] = (+)\lim_{x\to 3^-}\frac{1}{(x-3)^2} \overset{CV}{=} (+)\lim_{u\to 0^-}\frac{1}{u^2} \overset{S.1}{=} (+)(+\infty) \overset{E.4}{=} +\infty$$

Since $\lim\limits_{x\to 3^-} f(x) = +\infty$, $x = 3$ is a vertical asymptote.

$$\lim_{x\to(-4)^-} f(x) \overset{NS}{=} -\frac{4}{49}\left[\lim_{x\to(-4)^-}\frac{1}{(x+4)^{5/3}}\right] \overset{CV}{=} -\frac{4}{49}\lim_{u\to 0^-}\frac{1}{u^{5/3}} \overset{S.1}{=} (-)(-\infty) \overset{E.4}{=} +\infty$$

Since $\lim\limits_{x\to(-4)^-} f(x) = +\infty$, $x = -4$ is a vertical asymptote.

$$\lim_{x\to(-5)^-} f(x) \overset{NS}{=} \frac{5}{64}\left[\lim_{x\to(-5)^-}\frac{1}{\sqrt[5]{x+5}}\right] \overset{CV}{=} \frac{5}{64}\lim_{u\to 0^-}\frac{1}{\sqrt[5]{u}} \overset{S.1}{=} (+)(-\infty) \overset{E.4}{=} -\infty$$

Since $\lim\limits_{x\to(-5)^-} f(x) = -\infty$, $x = -5$ is a vertical asymptote.

Thus, $\boxed{f \text{ has **vertical asymptotes** at } x \in \{3, -4, -5\}}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{x}{(x-3)^2(x+4)^{5/3}\sqrt[5]{x+5}} = \lim_{x\to\infty}\frac{x^2}{x(x-3)^2(x+4)^{5/3}\sqrt[5]{x+5}} = \lim_{x\to\infty}\frac{\left(\frac{x}{x-3}\right)^2}{x(x+4)^{5/3}\sqrt[5]{x+5}}$$

$$\overset{L.3}{=} \lim_{x\to\infty}\left(\frac{x}{x-3}\right)^2 \lim_{x\to\infty}\frac{1}{x(x+4)^{5/3}\sqrt[5]{x+5}} = \lim_{x\to\infty}\left(\frac{1}{1-\frac{3}{x}}\right)^2 \lim_{x\to\infty}\frac{1}{x(x+4)^{5/3}\sqrt[5]{x+5}} = \left(\frac{1}{1-0}\right)^2 \lim_{x\to\infty}\frac{1}{x(x+4)^{5/3}\sqrt[5]{x+5}}$$

$$\overset{L.3}{=} \left[\lim_{x\to\infty}\frac{1}{x}\right]\left[\lim_{x\to\infty}\frac{1}{(x+4)^{5/3}}\right]\left[\lim_{x\to\infty}\frac{1}{\sqrt[5]{x+5}}\right] \overset{CV}{=} \left[\lim_{x\to\infty}\frac{1}{x}\right]\left[\lim_{u\to\infty}\frac{1}{u^{5/3}}\right]\left[\lim_{w\to\infty}\frac{1}{\sqrt[5]{w}}\right] = (0)(0)(0) = 0$$

Here's the above two **change of variables**:

$u = x+4 \iff x = u-4$ which means $x\to\infty \iff (u-4)\to\infty \iff u\to(\infty+4) \overset{E.2}{\iff} u\to\infty$

$w = x+5 \iff x = w-5$ which means $x\to\infty \iff (w-5)\to\infty \iff w\to(\infty+5) \overset{E.2}{\iff} w\to\infty$

Similarly, $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\dfrac{x}{(x-3)^2(x+4)^{5/3}\sqrt[5]{x+5}} = 0$

Thus, $\boxed{f \text{ has a **horizontal asymptote** at } y = 0}$

EXAMPLE: Let $h(x) = \dfrac{x^2+1}{x} = x + \dfrac{1}{x}$. Find all asymptotes.

h is undefined when $x = 0 \implies x = 0$ is **suspected** to be a **vertical asymptote**. Find out using the definition:

$$\lim_{x\to 0^-} h(x) = \lim_{x\to 0^-}\left[x + \frac{1}{x}\right] = \lim_{x\to 0^-} x + \lim_{x\to 0^-}\frac{1}{x} = 0 + (-\infty) = -\infty$$

Since **one** of the **1-sided limits** of f is **infinite**, $\boxed{h \text{ has a **vertical asymptote** at } x = 0}$.

$\lim\limits_{x\to-\infty} h(x) = -\infty$ and $\lim\limits_{x\to\infty} h(x) = \infty \implies \boxed{h \text{ has NO **horizontal asymptotes**}}$.

$\lim\limits_{x\to\infty}[h(x) - x] = \lim\limits_{x\to\infty}\dfrac{1}{x} = 0$ and $\lim\limits_{x\to-\infty}[h(x) - x] = \lim\limits_{x\to-\infty}\dfrac{1}{x} = 0 \implies \boxed{h \text{ has a **slant asymptote** } y = x}$.

EXAMPLE: Let $f(x) = 4 + \sqrt[3]{x} - \sqrt[3]{(x-8)^2}$

(a) Identify the **domain** of f.

$\text{Dom}(f) = \text{Dom}\left(4 + \sqrt[3]{x} - \sqrt[3]{(x-8)^2}\right) = \text{Dom}(4) \cap \text{Dom}(\sqrt[3]{x}) \cap \text{Dom}\left(\sqrt[3]{(x-8)^2}\right) = \mathbb{R} \cap \mathbb{R} \cap \mathbb{R} = \boxed{\mathbb{R} = (-\infty, \infty)}$

(b) Identify all **horizontal & vertical asymptotes** of f.

$\text{Dom}(f) = \mathbb{R} \implies f$ is undefined nowhere $\implies$ **no vertical asymptotes occur**.

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \left[4 + \sqrt[3]{x} - \sqrt[3]{(x-8)^2}\right] = (4+\infty)-\infty = \infty-\infty$ which is an **indeterminant form** (meaning inconclusive)

but notice as x gets more & more **positive**, $x < (x-8)^2 \implies \sqrt[3]{x} < \sqrt[3]{(x-8)^2}$.

Hence, the **3rd term dominates the others** as $x \to \infty \implies \lim_{x\to\infty} f(x) = -\infty \implies$ **no horizontal asymptote occurs**.

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \left[4 + \sqrt[3]{x} - \sqrt[3]{(x-8)^2}\right] = (4-\infty) - \infty = -\infty - \infty = -\infty \implies$ **no horizontal asymptote occurs**.

(c) Find all **critical numbers** of f which lie in the **interior** of its **domain** $\implies$ $\text{int}\,(\text{Dom}(f)) = \text{int}\,((-\infty,\infty)) = (-\infty,\infty)$

$f'(x) = \frac{d}{dx}\left[4 + x^{1/3} - (x-8)^{2/3}\right] = \frac{1}{3}x^{-2/3} - \frac{2}{3}(x-8)^{-1/3} = \frac{1}{3\sqrt[3]{x^2}} - \frac{2}{3\sqrt[3]{x-8}}$

$f'(x) \overset{set}{=} 0 \implies \frac{1}{3\sqrt[3]{x^2}} - \frac{2}{3\sqrt[3]{x-8}} = 0 \implies \sqrt[3]{x-8} - 2\sqrt[3]{x^2} = 0 \implies x - 8 = 8x^2 \implies 8x^2 - x + 8 = 0 \implies$ NO SOLN

Now, consider where $f'(x)$ is undefined yet $f(x)$ is defined: Division by zero occurs at $x \in \{0, 8\}$

Thus, $\boxed{\text{the critical numbers of } f \text{ are } x \in \{0, 8\}}$

(d) Identify all **relative minima, relative maxima** as well as the interval(s) where f **increases** and **decreases**.

In order to identify all this, **build the slope table** for f: [blue = critical #, red = vertical asymptote (VA)]

x	-19	0	7	8	9
$f'(x)$	$+$	DNE	$+$	DNE	$-$
slope	/	|	/	|	\

$\implies \boxed{f \text{ has a relative max (cusp) at } x = 8}$, $x = 0$ is NOT a relative extremum.

Relative extrema & **vertical asymptotes** separate intervals of increasing/decreasing:

$\boxed{f \text{ increases over } (-\infty, 8) \text{ OR } (-\infty, 0) \cup (0, 8),\ f \text{ decreases over } (8, \infty)}$

(e) Find all **CFIP's** (candidates for inflection point) of f which lie in the **interior** of its **domain**.

$f''(x) = \frac{d}{dx}\left[\frac{1}{3}x^{-2/3} - \frac{2}{3}(x-8)^{-1/3}\right] = -\frac{2}{9}x^{-5/3} + \frac{2}{9}(x-8)^{-4/3} = -\frac{2}{9\sqrt[3]{x^5}} + \frac{2}{9\sqrt[3]{(x-8)^4}}$

$f''(x) \overset{set}{=} 0 \implies -\frac{2}{9\sqrt[3]{x^5}} + \frac{2}{9\sqrt[3]{(x-8)^4}} = 0 \implies -\sqrt[3]{(x-8)^4} + \sqrt[3]{x^5} = 0 \implies x^5 = (x-8)^4 \implies x \approx 3.3937$

Now, consider where $f''(x)$ is undefined: Division by zero occurs at $x \in \{0, 8\}$ and they're NOT **vertical asymptotes**.

Thus, $\boxed{\text{the CFIP's of } f \text{ are } x \in \{0, 3.3937, 8\}}$

(f) Identify all **inflection points** as well as the interval(s) where f is **concave up** and **concave down**.

In order to identify all this, **build the concavity table** for f: [blue = CFIP, red = vertical asymptote (VA)]

x	-1	0	1	3.3937	7	8	9
$f''(x)$	$+$	DNE	$-$	0	$+$	DNE	$+$
concavity	$\cup$	$*$	$\cap$	$*$	$\cup$	$*$	$\cup$

$\implies \boxed{f \text{ has inflection points at } x \in \{0, 3.3937\}}$

Inflection points & **vertical asymptotes** separate intervals of concave up/concave down:

$\boxed{f \text{ is concave up (smiles) over } (-\infty, -1) \cup (3.3937, \infty),\ f \text{ is concave down (frowns) over } (0, 3.3937)}$

(g) Identify all **vertical tangents** and **cusps** of f.

Notice $x \in \{0, 8\}$ are both critical numbers and CFIP's.

Interpreting the 3rd rows of both tables implies that $\boxed{f \text{ has a } \textbf{vertical tangent} \text{ at } x = 0}$ and $\boxed{f \text{ has a } \textbf{cusp} \text{ at } x = 8}$

(h) Sketch the graph of f.

First, plot all **vertical & horizontal asymptotes** as **dashed lines**.

Then, plot the following points: hills, valleys, cusps, vertical tangents, inflection points.

Finally, complete the curve sketch using the guidance of the **third rows** of the **slope** & **concavity tables** of f.

EXAMPLE: Let $g(t) = \frac{1}{t^2 - t} = \frac{1}{t(t-1)}$

(a) Identify the **domain** of g.

$$\text{Dom}(g) = \text{Dom}\left(\frac{1}{t^2-t}\right) \overset{DM.Q}{=} \text{Dom}(1) \setminus \{t \in \mathbb{R} : t^2 - t = 0\} \overset{DM.1}{=} \boxed{\mathbb{R} \setminus \{0,1\} = (-\infty,0) \cup (0,1) \cup (1,\infty)}$$

(b) Identify all **horizontal & vertical asymptotes** of g.

$\text{Dom}(g) = \mathbb{R} \setminus \{0,1\} \implies g$ **may** have **vertical asymptotes** at $t = 0$ and $t = 1$. Use the definition to find out for sure:

$$\lim_{t\to0^+} g(t) = \lim_{t\to0^+} \frac{1}{t(t-1)} \overset{L.3}{=} \left[\lim_{t\to0^+} \frac{1}{t}\right]\left[\lim_{t\to0^+} \frac{1}{t-1}\right] \overset{NS}{=} \frac{1}{(0)-1}\left[\lim_{t\to0^+} \frac{1}{t}\right] \overset{S.1}{=} (-1)(+\infty) = -\infty$$

$$\lim_{t\to1^-} g(t) = \lim_{t\to1^-} \frac{1}{t(t-1)} \overset{L.3}{=} \left[\lim_{t\to1^-} \frac{1}{t}\right]\left[\lim_{t\to1^-} \frac{1}{t-1}\right] \overset{NS}{=} \frac{1}{(1)}\left[\lim_{t\to1^-} \frac{1}{t-1}\right] \overset{CV}{=} \lim_{u\to0^-} \frac{1}{u} \overset{S.1}{=} -\infty \qquad \Big(\text{where } u = t-1\Big)$$

Thus, by definition, indeed $\boxed{t = 0 \text{ and } t = 1 \text{ are } \textbf{vertical asymptotes} \text{ of } g}$.

$$\lim_{t\to\infty} g(t) = \lim_{t\to\infty} \frac{1}{t(t-1)} \overset{L.3}{=} \left[\lim_{t\to\infty} \frac{1}{t}\right]\left[\lim_{t\to\infty} \frac{1}{t-1}\right] \overset{CV}{=} \left[\lim_{t\to\infty} \frac{1}{t}\right]\left[\lim_{u\to\infty} \frac{1}{u}\right] \overset{S.1}{=} (0)(0) = 0 \qquad \Big(\text{where } u = t-1\Big)$$

$$\lim_{t\to-\infty} g(t) = \lim_{t\to-\infty} \frac{1}{t(t-1)} \overset{L.3}{=} \left[\lim_{t\to-\infty} \frac{1}{t}\right]\left[\lim_{t\to-\infty} \frac{1}{t-1}\right] \overset{CV}{=} \left[\lim_{t\to-\infty} \frac{1}{t}\right]\left[\lim_{u\to-\infty} \frac{1}{u}\right] \overset{S.1}{=} (0)(0) = 0 \qquad \Big(\text{where } u = t-1\Big)$$

Thus, $\boxed{y = 0 \text{ is the only } \textbf{horizontal asymptote} \text{ of } g}$.

(c) Find all **critical numbers** of g which lie in the **interior** of its **domain** $\implies$ $\text{int}(\text{Dom}(g)) = \text{int}(\mathbb{R} \setminus \{0,1\}) = \mathbb{R} \setminus \{0,1\}$

$$g'(t) = \frac{d}{dt}\left[\frac{1}{t^2-t}\right] \overset{D.4}{=} -\frac{2t-1}{(t^2-t)^2}, \qquad g'(t) \overset{set}{=} 0 \implies -\frac{2t-1}{(t^2-t)^2} = 0 \implies 2t - 1 = 0 \implies t = \frac{1}{2}$$

Consider where $g'(t)$ is undefined yet $g(t)$ is defined: Division by zero occurs at $t \in \{0,1\}$, but $g(0)$ and $g(1)$ are **undefined**.

Thus, $\boxed{\text{the only critical number of } g \text{ is } t = \frac{1}{2}}$

(d) Identify all **relative minima, relative maxima** as well as the interval(s) where f **increases** and **decreases**.

In order to identify all this, **build the slope table** for g: [blue = critical #, red = vertical asymptote (VA)]

t	-1	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	2
$g'(t)$	$+$	DNE	$+$	0	$-$	DNE	$-$
slope	/	VA	/	$-$	\	VA	\

$\implies \boxed{g \text{ has a relative max (hill) at } t = \frac{1}{2}}$.

Relative extrema & **vertical asymptotes** separate intervals of increasing/decreasing:

$$\boxed{g \text{ increases over } (-\infty,0) \cup \left(0,\frac{1}{2}\right), \; g \text{ decreases over } \left(\frac{1}{2},1\right) \cup (1,\infty)}$$

(e) Find all **CFIP's** (candidates for inflection point) of g which lie in the **interior** of its **domain**.

$$g''(t) = \frac{d}{dt}\left[-\frac{2t-1}{(t^2-t)^2}\right] \overset{D.4}{=} \frac{6t^2-6t+2}{(t-1)^3t^3}, \qquad g''(t) \overset{set}{=} 0 \implies \frac{6t^2-6t+2}{(t-1)^3t^3} = 0 \implies 6t^2 - 6t + 2 = 0 \implies \text{NO SOLN}$$

Now, consider where $g''(t)$ is undefined: Division by zero only occurs at **vertical asymptotes** $\implies$ $\boxed{g \text{ has NO CFIP's}}$

(f) Identify all **inflection points** as well as the interval(s) where g is **concave up** and **concave down**.

In order to identify all this, **build the concavity table** for g: [blue = CFIP, red = vertical asymptote (VA)]

t	-10	0	$\frac{1}{2}$	1	10
$g''(t)$	$+$	DNE	$-$	DNE	$+$
concavity	$\cup$	VA	$\cap$	VA	$\cup$

$\implies \boxed{g \text{ has NO inflection points since there's NO CFIP's}}$

Inflection points & **vertical asymptotes** separate intervals of concave up/concave down:

$$\boxed{g \text{ is concave up (smiles) over } (-\infty,0) \cup (1,\infty), \; f \text{ is concave down (frowns) over } (0,1)}$$

(g) Identify all **vertical tangents** and **cusps** of g.

Interpreting the 3rd rows of both tables implies that $\boxed{g \text{ has NO vertical tangents or cusps}}$.

(h) Sketch the graph of g.

Plot all vertical asymptotes, horizontal asymptotes, hills, valleys, cusps, vertical tangents, and inflection points.

Complete the curve sketch using the guidance of the **third rows** of the **slope** & **concavity tables** of g.