FINITE SUMS

SUMMATION RULES: $\left(m, n \in \mathbb{Z} \text{ and } a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n, c \in \mathbb{R}\right)$

- (FS.0) (Constant Rule) $\displaystyle\sum_{k=1}^{n} c = nc$
- (FS.1) (Constant Multiple) $\displaystyle\sum_{k=1}^{n} ca_k = ca_1 + ca_2 + ca_3 + \cdots + ca_{n-2} + ca_{n-1} + ca_n = c\sum_{k=1}^{n} a_k$
- (FS.2) (Sum/Diff Rule) $\displaystyle\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$
- (FS.3) (Lump-Sum Rule) $1 < m < n \implies \displaystyle\sum_{k=1}^{m} a_k + \sum_{k=m+1}^{n} a_k = \sum_{k=1}^{n} a_k$
- (FS.4) (Sum of Integers) $\displaystyle\sum_{i=1}^{n} i = 1 + 2 + 3 + 4 + \cdots + (n-2) + (n-1) + n = \frac{n(n+1)}{2}$
- (FS.5) (Sum of Squares) $\displaystyle\sum_{j=1}^{n} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + \cdots + (n-2)^2 + (n-1)^2 + n^2 = \frac{n(n+1)(2n+1)}{6}$
- (FS.6) (Sum of Cubes) $\displaystyle\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + (n-2)^3 + (n-1)^3 + n^3 = \frac{n^2(n+1)^2}{4}$

REMARKS:

- The **index** of the sum can be labeled however one pleases: $\displaystyle\sum_{i=2}^{10}(1+i^3) = \sum_{j=2}^{10}(1+j^3) = \sum_{\alpha=2}^{10}(1+\alpha^3) = 3033$
- Finite sums are necessary in the construction of **Riemann sums**.
- **Infinite sums** such as $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ are addressed in Calculus II.

EXAMPLE: Expand (but not compute) the sum: $\displaystyle\sum_{j=-2}^{5} \cos j$

$$\sum_{j=-2}^{5} \cos j = \boxed{\cos(-2) + \cos(-1) + \cos 0 + \cos 1 + \cos 2 + \cos 3 + \cos 4 + \cos 5}$$

EXAMPLE: Evaluate the sum: $\displaystyle\sum_{\substack{k=1 \\ k \text{ odd}}}^{10} k^2 e^k$

$$\sum_{\substack{k=1 \\ k \text{ odd}}}^{10} k^2 e^k = (1)^2 e^{(1)} + (3)^2 e^{(3)} + (5)^2 e^{(5)} + (7)^2 e^{(7)} + (9)^2 e^{(9)} = \boxed{e + 9e^3 + 25e^5 + 49e^7 + 81e^9 \approx 713978.64}$$

EXAMPLE: Evaluate the sum: $\displaystyle\sum_{m=0}^{4} \frac{m+1}{m+2}$

$$\sum_{m=0}^{4} \frac{m+1}{m+2} = \frac{(0)+1}{(0)+2} + \frac{(1)+1}{(1)+2} + \frac{(2)+1}{(2)+2} + \frac{(3)+1}{(3)+2} + \frac{(4)+1}{(4)+2} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} = \boxed{\frac{71}{20} = 3.55}$$

EXAMPLE: Evaluate the sum: $\displaystyle\sum_{k=1}^{20}(2k^2 - 5k + 3)$

$$\sum_{k=1}^{20}(2k^2 - 5k + 3) \overset{FS.2}{=} \sum_{k=1}^{20} 2k^2 - \sum_{k=1}^{20} 5k + \sum_{k=1}^{20} 3 \overset{FS.1}{=} 2\sum_{k=1}^{20} k^2 - 5\sum_{k=1}^{20} k + \sum_{k=1}^{20} 3 = 2\left[\frac{(20)(21)(41)}{6}\right] - 5\left[\frac{(20)(21)}{2}\right] + 3(20)$$

$$= 5740 - 1050 + 60 = \boxed{4750}$$
