INDEFINITE INTEGRATION

INTRODUCTION:

- A function $F(x)$ is the **anti-derivative** of $f(x)$ on interval $I \iff \int f(x)\,dx = F(x)\ \forall x \in I \iff F'(x) = f(x)\ \forall x \in I$.
- The **indefinite integral** of $f(x)$ is denoted $\int f(x)\,dx$ and is read: "The integral of $f(x)$ with respect to x"
- Geometrically, $\int f(x)\,dx$ is a **family of curves** $F(x) + C$ such that the **slope** at x is $f(x)\quad \forall C \in \mathbb{R}$

ONE-DIMENSIONAL MOTION OF A PARTICLE:

- Given **velocity function** $v(t)$, the **position function** is $s(t) = \int v(t)\,dt + C$
- Given **acceleration function** $a(t)$, the **velocity function** is $v(t) = \int a(t)\,dt + C$

BASIC INDEFINITE INTEGRAL RULES:

Here, $C \in \mathbb{R}$ is the **constant of integration**.

- (INT.0) (Zero Rule) $\int 0\,dx = C$
- (INT.1) (Constant Rule) $\int k\,dx = kx + C$ $\big($where $k \in \mathbb{R}\big)$
- (INT.2) (Multiple Rule) $\int kf(x)\,dx = k\int f(x)\,dx$ $\big($where $k \in \mathbb{R}\big)$
- (INT.3) (Sum/Diff Rule) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- (INT.4) (Power Rule) $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ $\big($provided $n \in \mathbb{R} \setminus \{-1\}\big)$
- (INT.5) (Natural Exp) $\int e^x\,dx = e^x + C$
- (INT.6) (General Exp) $\int a^x\,dx = \frac{a^x}{\ln a} + C$ $\big($provided $a \in \mathbb{R}_+ \setminus \{1\}\big)$
- (INT.7) (Reciprocal) $\int \frac{1}{x}\,dx = \ln|x| + C$
- (INT.8) (Sine Rule) $\int \sin x\,dx = -\cos x + C$ (INT.9) (Cosine Rule) $\int \cos x\,dx = \sin x + C$
- (INT.10) (Secant-squared) $\int \sec^2 x\,dx = \tan x + C$ (INT.11) (Secant-Tangent) $\int \sec x \tan x\,dx = \sec x + C$
- (INT.12) (Cosecant-squared) $\int \csc^2 x\,dx = -\cot x + C$ (INT.13) (Cosecant-Cotangent) $\int \csc x \cot x\,dx = -\csc x + C$
- (INT.14) (Inv Sine) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$
- (INT.15) (Inv Tangent) $\int \frac{1}{1+x^2}\,dx = \arctan x + C$
- (INT.16) (Inv Secant) $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x + C$

REMARKS:

- Functions of arbitrary constants yield arbitrary contants. (e.g. $C_1 + C_2 = C_3, \sqrt{C_1} = C_4, e^{C_1} = C_5$, ...)
- There's no general "product rule" or "quotient rule" for integration.
- Some 'simple-looking' integrals are not. They will be covered in Calculus II: $\int \ln x\,dx, \int \cos\sqrt{x}\,dx, \int \arcsin x\,dx, \ldots$
- (**nonelementary integral**) Some integrals are not even elementary! These are also addressed in Calculus II: $\int e^{x^2}\,dx, \int \frac{e^x}{x}\,dx, \int \sin(x^2)\,dx, \int \cos(e^x)\,dx, \int \sqrt{1+x^4}\,dx, \int \ln(\ln x)\,dx, \int \frac{1}{\ln x}\,dx, \int \frac{\sin x}{x}\,dx, \int x^x\,dx, \ldots$