INTEGRATION: STRATEGY [SST 7.5]

RELEVANT TRIG IDENTITIES: $\Big($Memorize these$\Big)$

$\sin\theta := \frac{OPP}{HYP}, \cos\theta := \frac{ADJ}{HYP}, \tan\theta := \frac{OPP}{ADJ}, \csc\theta := \frac{HYP}{OPP}, \sec\theta := \frac{HYP}{ADJ}, \cot\theta := \frac{ADJ}{OPP}$

$\csc\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \cot\theta = \frac{1}{\tan\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1, \tan^2\theta + 1 = \sec^2\theta, 1 + \cot^2\theta = \csc^2\theta$

$\sin(-\theta) = -\sin\theta, \cos(-\theta) = \cos\theta$

$\sin(A+B) = \sin A\cos B + \cos A\sin B, \cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B, \cos(A-B) = \cos A\cos B + \sin A\sin B$

$\sin(2\theta) = 2\sin\theta\cos\theta, \cos^2\theta = \frac{1+\cos(2\theta)}{2}, \sin^2\theta = \frac{1-\cos(2\theta)}{2}$

BASIC INTEGRAL RULES: $\Big($Memorize these$\Big)$

$\int x^n\ dx$	$= \frac{x^{n+1}}{n+1} + C$	$\int \frac{1}{x}\ dx$	$= \ln\lvert x\rvert + C$
$\int e^x\ dx$	$= e^x + C$	$\int a^x\ dx$	$= \frac{a^x}{\ln a} + C$
$\int \sin x\ dx$	$= -\cos x + C$	$\int \cos x\ dx$	$= \sin x + C$
$\int \sec^2 x\ dx$	$= \tan x + C$	$\int \csc^2 x\ dx$	$= -\cot x + C$
$\int \sec x\tan x\ dx$	$= \sec x + C$	$\int \csc x\cot x\ dx$	$= -\csc x + C$
$\int \tan x\ dx$	$= \ln\lvert\sec x\rvert + C$	$\int \cot x\ dx$	$= \ln\lvert\sin x\rvert + C$
$\int \sec x\ dx$	$= \ln\lvert\sec x + \tan x\rvert + C$	$\int \csc x\ dx$	$= -\ln\lvert\csc x + \cot x\rvert + C$

INTEGRATION TOOLBOX:

- Algebraic Simplification
- Factoring: Monomial, by Grouping, Sum of Cubes, Difference of Squares/Cubes, ...
- Binomial Theorem/Pascal's Triangle
- Rationalizing the Numerator (RN)
- Rationalizing the Denominator (RD)
- Clever insertion of one (CI-1)
- Clever insertion of zero (CI-0)
- Split Fraction (SF)
- Trig Identities (TRG)
- Reference Triangles
- Basic Integral Rules (see above)
- Change of Variables (CV)
- Integration by Parts (IBP)
- Partial Fraction Decomposition (PFD)
- Completing the Square (CS)
- Forms involving powers of sine and/or cosine
- Forms involving powers of secant and/or tangent
- Forms involving powers of cosecant and/or cotangent
- Forms involving $\sqrt{a^2-u^2}$, $\sqrt{a^2+u^2}$, or $\sqrt{u^2-a^2}$
- Forms involving $\dfrac{1}{ax^2+bx+c}$ or $\dfrac{1}{\sqrt{ax^2+bx+c}}$
- $\int \sin(mx)\sin(nx)\ dx$, $\int \sin(mx)\cos(nx)\ dx$, $\int \cos(mx)\cos(nx)\ dx$ $\Big($where $n, m \in \mathbb{Z}\setminus\{0\}\Big)$

EX 7.5.1: Evaluate $I = \int_{\sqrt{\pi}}^{\sqrt{\pi}} e^{x^x} \sin(\cos(\ln x)) \ dx$.

EX 7.5.2: Evaluate $I = \int \frac{1}{1+\frac{1}{x}} \ dx$.

EX 7.5.3: Evaluate $I = \int \ln(\sqrt[7]{y}) \ dy$.

EX 7.5.4: Evaluate $I = \int e^{\sqrt[3]{t}} \ dt$.

EX 7.5.5: Evaluate $I = \int \frac{x^5}{x^{12}+9}\,dx$.

EX 7.5.6: Evaluate $I = \int \frac{x^2}{x^6-a^6}\,dx$, where $a \in \mathbb{R}$.

EX 7.5.7: Evaluate $I = \int \frac{1}{e^x - e^{-x}}\,dx$.

EX 7.5.8 Evaluate $I = \int \sqrt{\frac{1-x}{1+x}}\, dx$.

EX 7.5.9 Evaluate $I = \int \frac{x^3}{(x-1)^{100}}\, dx$.

EX 7.5.10 Evaluate $I = \int \frac{x \ln x}{\sqrt{x^2 - 1}} \, dx$.

EX 7.5.11 Evaluate $I = \int \frac{\ln(x+1)}{x^2} \, dx$.

EX 7.5.12 Evaluate $I = \int \dfrac{1}{\sqrt{x+2}+\sqrt{x}}\, dx$.

EX 7.5.13 Evaluate $I = \int \dfrac{\sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$.