

VECTORS IN \mathbb{R}^2 [SST 9.1]

VECTORS & SCALARS:

- A **2-D vector** $\mathbf{v} \in \mathbb{R}^2$ is a quantity that **has both magnitude and direction**. (Hand-write vector \mathbf{v} as \vec{v})
 - e.g. displacement, velocity, force, angular momentum, ...
- A **scalar** $s \in \mathbb{R}$ is a quantity that **only has magnitude**.
 - e.g. time, temperature, distance, speed, volume, electric charge, ...

VECTOR NOTATION:

- **Component form** of a vector: $\mathbf{v} = \langle v_1, v_2 \rangle$
 - * v_1 is the **horizontal component** of vector \mathbf{v} .
 - * v_2 is the **vertical component** of vector \mathbf{v} .
- **Standard basis form** of a vector: $\mathbf{v} = v_1\hat{\mathbf{i}} + v_2\hat{\mathbf{j}}$ or $\mathbf{v} = v_1\hat{\mathbf{e}}_1 + v_2\hat{\mathbf{e}}_2$
 - * $\hat{\mathbf{i}} = \hat{\mathbf{e}}_1 = \langle 1, 0 \rangle$
 - * $\hat{\mathbf{j}} = \hat{\mathbf{e}}_2 = \langle 0, 1 \rangle$
- The vector starting at point $P(x_1, y_1)$ and ending at point $Q(x_2, y_2)$ is $\mathbf{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$
 - Point $P(x_1, y_1)$ is called the **initial point of vector PQ**
 - Point $Q(x_2, y_2)$ is called the **terminal point of vector PQ**
- Vectors \mathbf{u}, \mathbf{v} are **equal** if and only if their components are equal: $u_1 = v_1$ and $u_2 = v_2$.
- **Zero vector**: $\mathbf{0} := \langle 0, 0 \rangle$
- The vector **opposite of vector** \mathbf{v} is $-\mathbf{v}$.
- The **norm** of a vector \mathbf{v} , denoted $\|\mathbf{v}\|$, is the length of the vector.
- A **unit vector**, denoted $\hat{\mathbf{v}}$, is a vector with **norm one**.
- A **direction vector** for a nonzero vector \mathbf{v} is a unit vector with the same direction as \mathbf{v} .

BASIC OPERATIONS WITH VECTORS:

- Vector Addition: $\mathbf{u} + \mathbf{v} = \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$
- Vector Subtraction: $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \langle u_1 - v_1, u_2 - v_2 \rangle$
- Scalar Multiplication: $t\mathbf{v} = t\langle v_1, v_2 \rangle = \langle tv_1, tv_2 \rangle$
- Norm of a Vector: $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$
- Direction Vector: $\hat{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$
- Linear Combination of Two Vectors: $s\mathbf{u} + t\mathbf{v} = \langle su_1 + tv_1, su_2 + tv_2 \rangle$
- **BEWARE:** The notion of "multiplying or dividing two vectors" is NOT DEFINED!!

PROPERTIES OF VECTORS:

(Let vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ and scalars $s, t \in \mathbb{R}$)

Vector Commutativity:	$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	Vector Associativity:	$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
Additive Identity:	$\mathbf{u} + \mathbf{0} = \mathbf{u}$	Additive Inverse:	$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
Vector Distribution:	$(s + t)\mathbf{u} = s\mathbf{u} + t\mathbf{u}$	Scalar Distribution:	$s(\mathbf{u} + \mathbf{v}) = s\mathbf{u} + s\mathbf{v}$
Scalar Multiplication Associativity:	$(st)\mathbf{u} = s(t\mathbf{u})$		

EX 9.1.1: Let $\mathbf{u} = \langle 1, 2 \rangle$, $\mathbf{v} = \langle 4, -5 \rangle$, and $\mathbf{w} = -7\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$.

(a) Compute $3(\mathbf{u} - \mathbf{w}) + \frac{1}{2}\mathbf{v}$.

(b) Compute $\|\mathbf{u}\| + \|\mathbf{v}\|^2 - 3\|\mathbf{w}\|^2$.

(c) Find the **terminal point** of vector \mathbf{u} if the **initial point** is $(-2, 3)$.

(d) Find the unit vector $\hat{\mathbf{u}}$.

(e) Find the **unit vector** that is in the **same direction** as $\langle 10, 5 \rangle$.

(f) Find the **unit vector** that is in the **opposite direction** as $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$.

(g) Find the vector of norm 4 that is 30° counterclockwise from the positive x -axis.

(h) Find scalars s and t such that $s\langle -1, 1 \rangle + t(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = \mathbf{w}$
