VECTORS IN $\mathbb{R}^3$ [SST 9.2]

VECTORS & SCALARS:

- A **3-D vector** $\mathbf{v} \in \mathbb{R}^3$ is a quantity that **has both magnitude and direction**. $\left(\text{Hand-write vector } \mathbf{v} \text{ as } \vec{v}\right)$
  - e.g. displacement, velocity, force, angular momentum, ...
- A **scalar** $s \in \mathbb{R}$ is a quantity that **only has magnitude**.
  - e.g. time, temperature, distance, speed, volume, electric charge, ...

VECTOR NOTATION:

- **Component form** of a vector: $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$
- **Standard basis form** of a vector: $\mathbf{v} = v_1\widehat{\mathbf{i}} + v_2\widehat{\mathbf{j}} + v_3\widehat{\mathbf{k}}$ or $\mathbf{v} = v_1\widehat{\mathbf{e}}_1 + v_2\widehat{\mathbf{e}}_2 + v_3\widehat{\mathbf{e}}_3$
  - ⋆ $\widehat{\mathbf{i}} = \widehat{\mathbf{e}}_1 = \langle 1, 0, 0 \rangle$
  - ⋆ $\widehat{\mathbf{j}} = \widehat{\mathbf{e}}_2 = \langle 0, 1, 0 \rangle$
  - ⋆ $\widehat{\mathbf{k}} = \widehat{\mathbf{e}}_3 = \langle 0, 0, 1 \rangle$
- The vector starting at point $P(x_1, y_1, z_1)$ and ending at point $Q(x_2, y_2, z_2)$ is $\mathbf{PQ} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$
  - Point $P(x_1, y_1, z_1)$ is called the **initial point of vector PQ**
  - Point $Q(x_2, y_2, z_2)$ is called the **terminal point of vector PQ**
- Vectors $\mathbf{u}, \mathbf{v}$ are **equal** if and only if their components are equal: $u_1 = v_1$ and $u_2 = v_2$ and $u_3 = v_3$.
- **Zero vector**: $\mathbf{0} := \langle 0, 0, 0 \rangle$
- The vector **opposite of vector v** is $-\mathbf{v}$.
- The **norm** of a vector $\mathbf{v}$, denoted $||\mathbf{v}||$, is the length of the vector.
- A **unit vector**, denoted $\widehat{\mathbf{v}}$, is a vector with **norm one**.
- A **direction vector** for a nonzero vector $\mathbf{v}$ is a unit vector with the same direction as $\mathbf{v}$.

BASIC OPERATIONS WITH VECTORS:

- Vector Addition: $\mathbf{u} + \mathbf{v} = \langle u_1, u_2, u_3 \rangle + \langle v_1, v_2, v_3 \rangle = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$
- Vector Subtraction: $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle$
- Scalar Multiplication: $t\mathbf{v} = t\langle v_1, v_2, v_3 \rangle = \langle tv_1, tv_2, tv_3 \rangle$
- Norm of a Vector: $||\mathbf{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
- Direction Vector: $\widehat{\mathbf{v}} = \dfrac{\mathbf{v}}{||\mathbf{v}||}$
- Linear Combination of Two Vectors: $s\mathbf{u} + t\mathbf{v} = \langle su_1 + tv_1, su_2 + tv_2, su_3 + tv_3 \rangle$
- BEWARE: The notion of "multiplying or dividing two vectors" is NOT DEFINED!!

PROPERTIES OF VECTORS: $\left(\text{Let vectors } \mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3 \text{ and scalars } s, t \in \mathbb{R}\right)$

Vector Commutativity:	$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	Vector Associativity:	$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
Additive Identity:	$\mathbf{u} + \mathbf{0} = \mathbf{u}$	Additive Inverse:	$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
Vector Distribution:	$(s + t)\mathbf{u} = s\mathbf{u} + t\mathbf{u}$	Scalar Distribution:	$s(\mathbf{u} + \mathbf{v}) = s\mathbf{u} + s\mathbf{v}$
Scalar Multiplication Associativity:	$(st)\mathbf{u} = s(t\mathbf{u})$		

EX 9.2.1: Let $\mathbf{u} = \langle 1, 2, 3 \rangle$, $\mathbf{v} = \langle 2, -1, -1 \rangle$, and $\mathbf{w} = -3\widehat{\mathbf{i}} - 3\widehat{\mathbf{j}} - \widehat{\mathbf{k}}$.

(a) Compute $3(\mathbf{u} - \mathbf{w}) + \frac{1}{2}\mathbf{v}$.

(b) Compute $||\mathbf{u}|| + ||\mathbf{v}||^2 - 3||\mathbf{w}||^2$.

(c) Find the **terminal point** of vector $\mathbf{u}$ if the **initial point** is $(-2, 3, -4)$.

(d) Find the unit vector $\widehat{\mathbf{u}}$.

(e) Find the **unit vector** that is in the **same direction** as $\langle 10, 5, -2 \rangle$.

(f) Find the **unit vector** that is in the **opposite direction** as $3\widehat{\mathbf{i}} - 5\widehat{\mathbf{j}} - \widehat{\mathbf{k}}$.