WeBWorK: HW 6.3 $\to$ Area Problems

Unfortunately, WeBWorK wants you to actually find the area of polar regions (not setup the integral).

For some area problems, you will run into integrals of the form:

$$\int_\alpha^\beta \cos^2\theta \; d\theta \quad \text{or} \quad \int_\alpha^\beta \sin^2\theta \; d\theta$$

You'll learn about similar such integrals in Chapter 7, but for now, here's the key to finding the antiderivative:

Use the trig identities: $\begin{cases} \cos^2\theta = \dfrac{1}{2}\left[1+\cos(2\theta)\right] \\[2ex] \sin^2\theta = \dfrac{1}{2}\left[1-\cos(2\theta)\right] \end{cases}$

Hence,

$$\boxed{\int \cos^2\theta \; d\theta = \int \frac{1}{2}\left[1+\cos(2\theta)\right] \; d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta)}$$

$$\boxed{\int \sin^2\theta \; d\theta = \int \frac{1}{2}\left[1-\cos(2\theta)\right] \; d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)}$$

In case you're wondering how the above trig identities came about, start with the **half-angle identities**:

$$\begin{cases} \cos\left(\dfrac{\phi}{2}\right) = \pm\sqrt{\dfrac{1+\cos\phi}{2}} \\[2ex] \sin\left(\dfrac{\phi}{2}\right) = \pm\sqrt{\dfrac{1-\cos\phi}{2}} \end{cases} \implies \begin{cases} \cos^2\left(\dfrac{\phi}{2}\right) = \dfrac{1+\cos\phi}{2} \\[2ex] \sin^2\left(\dfrac{\phi}{2}\right) = \dfrac{1-\cos\phi}{2} \end{cases} \overset{CV}{\implies} \begin{cases} \cos^2\theta = \dfrac{1+\cos(2\theta)}{2} \\[2ex] \sin^2\theta = \dfrac{1-\cos(2\theta)}{2} \end{cases} \quad \text{(CV) } \theta := \frac{\phi}{2} \iff \phi = 2\theta$$