EX 3.2.2: Using elementary row/column operations as appropriate, find the determinant of $A = \begin{bmatrix} -12 & 85 & 60 & 18 \\ -1 & 7 & 5 & 3 \\ 3 & -24 & -21 & 12 \\ 4 & -27 & -17 & -13 \end{bmatrix}$.

Perform row operations until either a triangular matrix or matrix with a row or column of all zeros is attained:

$$|A| \xrightarrow{R_1 \leftrightarrow R_2} (-1)\begin{vmatrix} -1 & 7 & 5 & 3 \\ -12 & 85 & 60 & 18 \\ 3 & -24 & -21 & 12 \\ 4 & -27 & -17 & -13 \end{vmatrix} \xrightarrow[\substack{3R_1+R_3 \to R_3 \\ 4R_1+R_4 \to R_4}]{(-12)R_1+R_2 \to R_2} (-1)\begin{vmatrix} -1 & 7 & 5 & 3 \\ 0 & 1 & 0 & -18 \\ 0 & -3 & -6 & 21 \\ 0 & 1 & 3 & -1 \end{vmatrix}$$

$$\xrightarrow[(-1)R_2+R_4 \to R_4]{3R_2+R_3 \to R_3} (-1)\begin{vmatrix} -1 & 7 & 5 & 3 \\ 0 & 1 & 0 & -18 \\ 0 & 0 & -6 & -33 \\ 0 & 0 & 3 & 17 \end{vmatrix} \xrightarrow{2R_4 \to R_4} (-1)\left(\frac{1}{2}\right)\begin{vmatrix} -1 & 7 & 5 & 3 \\ 0 & 1 & 0 & -18 \\ 0 & 0 & -6 & -33 \\ 0 & 0 & 6 & 34 \end{vmatrix}$$

$$\xrightarrow{R_3+R_4 \to R_4} (-1)\left(\frac{1}{2}\right)\begin{vmatrix} -1 & 7 & 5 & 3 \\ 0 & 1 & 0 & -18 \\ 0 & 0 & -6 & -33 \\ 0 & 0 & 0 & 1 \end{vmatrix} \overset{(*)}{=} (-1)\left(\frac{1}{2}\right)\Big[(-1)(1)(-6)(1)\Big] = \boxed{-3}$$

(∗) The final matrix is **upper triangular**, hence its determinant is the **product of its main diagonal entries**.