EX 5.1.3: Let vectors $\mathbf{u} = (1,2,3,4)^T$ and $\mathbf{v} = (-3,3,0,-1)^T$.

(a) Compute dot product $\mathbf{v}^T\mathbf{u}$.

$$\mathbf{v}^T\mathbf{u} = \mathbf{v}\cdot\mathbf{u} = (-3)(1) + (3)(2) + (0)(3) + (-1)(4) = \boxed{-1}$$

(b) Are $\mathbf{u}$ & $\mathbf{v}$ orthogonal?

$\boxed{\text{No, since } \mathbf{v}\cdot\mathbf{u} \neq 0}$

(c) Compute $||\mathbf{v}||$.

$$||\mathbf{v}|| = \sqrt{(-3)^2 + 3^2 + 0^2 + (-1)^2} = \boxed{19}$$

(d) Find unit vector $\widehat{\mathbf{v}}$.

$$\widehat{\mathbf{v}} = \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{1}{\sqrt{19}}(-3,3,0,-1)^T = \boxed{\left(-\frac{3}{\sqrt{19}}, \frac{3}{\sqrt{19}}, 0, -\frac{1}{\sqrt{19}}\right)^T} \text{ OR } \boxed{\begin{bmatrix} -3/\sqrt{19} \\ 3/\sqrt{19} \\ 0 \\ -1/\sqrt{19} \end{bmatrix}}$$

(e) Compute orthogonal projections $\text{proj}_{\mathbf{u}}\mathbf{v}$ and $\text{proj}_{\mathbf{v}}\mathbf{u}$.

$$\text{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}} = \frac{(1)(-3)+(2)(3)+(3)(0)+(4)(-1)}{(1)(1)+(2)(2)+(3)(3)+(4)(4)}\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = \frac{-1}{30}\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = \boxed{\begin{bmatrix}-1/30\\-1/15\\-1/10\\-2/15\end{bmatrix}} \text{ OR } \boxed{\left(-\frac{1}{30}, -\frac{1}{15}, -\frac{1}{10}, -\frac{2}{15}\right)^T}$$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}} = \frac{(1)(-3)+(2)(3)+(3)(0)+(4)(-1)}{(-3)(-3)+(3)(3)+(0)(0)+(-1)(-1)}\begin{bmatrix}-3\\3\\0\\-1\end{bmatrix} = \frac{-1}{19}\begin{bmatrix}-3\\3\\0\\-1\end{bmatrix} = \boxed{\begin{bmatrix}3/19\\-3/19\\0\\1/19\end{bmatrix}} \text{ OR } \boxed{\left(\frac{3}{19}, -\frac{3}{19}, 0, \frac{1}{19}\right)^T}$$