EX 6.3.2: Find std matrix A for linear transformation $L:\mathbb{R}^2\to\mathbb{R}^4$ s.t. $L\left(\begin{bmatrix}1\\4\end{bmatrix}\right)=\begin{bmatrix}-3\\14\\-2\\6\end{bmatrix}$, $L\left(\begin{bmatrix}2\\-3\end{bmatrix}\right)=\begin{bmatrix}16\\-5\\7\\12\end{bmatrix}$

$$\left[\begin{array}{c|c} \text{---}\ \mathbf{b}_1\ \text{---} & L(\mathbf{b}_1) \\ \text{---}\ \mathbf{b}_2\ \text{---} & L(\mathbf{b}_2)\end{array}\right]=\left[\begin{array}{cc|c}\boxed{1} & 4 & (-3,14,-2,6)^T\\ 2 & -3 & (16,-5,7,12)^T\end{array}\right]\xrightarrow{(-2)R_1+R_2\to R_2}\left[\begin{array}{cc|c}\boxed{1} & 4 & (-3,14,-2,6)^T\\ 0 & -11 & (22,-33,11,0)^T\end{array}\right]$$

$$\xrightarrow{(-\frac{1}{11})R_2\to R_2}\left[\begin{array}{cc|c}\boxed{1} & 4 & (-3,14,-2,6)^T\\ 0 & \boxed{1} & (-2,3,-1,0)^T\end{array}\right]\xrightarrow{(-4)R_2+R_1\to R_1}\left[\begin{array}{cc|c}\boxed{1} & 0 & (5,2,2,6)^T\\ 0 & \boxed{1} & (-2,3,-1,0)^T\end{array}\right]=\left[\begin{array}{c|c} I & \begin{matrix}L(\mathbf{e}_1)\\ L(\mathbf{e}_2)\end{matrix}\end{array}\right]$$

$$\implies A=\begin{bmatrix}| & |\\ L(\mathbf{e}_1) & L(\mathbf{e}_2)\\ | & |\end{bmatrix}=\boxed{\begin{bmatrix}5 & -2\\ 2 & 3\\ 2 & -1\\ 6 & 0\end{bmatrix}}\qquad \therefore\ L\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{bmatrix}5 & -2\\ 2 & 3\\ 2 & -1\\ 6 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}5x_1-2x_2\\ 2x_1+3x_2\\ 2x_1-x_2\\ 6x_1\end{bmatrix}$$

EX 6.3.3: Find standard matrix A for linear transformation $L:P_2\to P_2$ s.t. $L(p)=t^2p''(t)$.

Find the images of the **standard basis vectors** for the **domain** of L: (Write polynomials in **ascending** order)

$$\begin{array}{lllllll} L(\mathbf{e}_1) & = & L(1) & = & t^2\frac{d^2}{dt^2}[1] & = 0 & \simeq (0,0,0)^T\\ L(\mathbf{e}_2) & = & L(t) & = & t^2\frac{d^2}{dt^2}[t] & = 0 & \simeq (0,0,0)^T\\ L(\mathbf{e}_3) & = & L(t^2) & = & t^2\frac{d^2}{dt^2}[t^2] & = 2t^2 & \simeq (0,0,2)^T\end{array}\implies A=\boxed{\begin{bmatrix}| & | & |\\ L(\mathbf{e}_1) & L(\mathbf{e}_2) & L(\mathbf{e}_3)\\ | & | & |\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&2\end{bmatrix}}$$

$$\therefore\ L(a+bt+ct^2)\simeq\begin{bmatrix}0&0&0\\0&0&0\\0&0&2\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}0\\0\\2c\end{bmatrix}\simeq 2ct^2$$

EX 6.3.4: Find standard matrix A for linear transformation $L:P_2\to P_3$ s.t. $L(p)=\int p(t)\,dt$. (Do not include '+C')

Find the images of the **standard basis vectors** for the **domain** of L: (Write polynomials in **ascending** order)

$$\begin{array}{lllllll} L(\mathbf{e}_1) & = & L(1) & = & \int 1\,dt & = t & \simeq (0,1,0,0)^T\\ L(\mathbf{e}_2) & = & L(t) & = & \int t\,dt & = \frac{1}{2}t^2 & \simeq (0,0,\frac{1}{2},0)^T\\ L(\mathbf{e}_3) & = & L(t^2) & = & \int t^2\,dt & = \frac{1}{3}t^3 & \simeq (0,0,0,\frac{1}{3})^T\end{array}\implies A=\boxed{\begin{bmatrix}| & | & |\\ L(\mathbf{e}_1) & L(\mathbf{e}_2) & L(\mathbf{e}_3)\\ | & | & |\end{bmatrix}=\begin{bmatrix}0&0&0\\1&0&0\\0&1/2&0\\0&0&1/3\end{bmatrix}}$$

$$\therefore\ L(a+bt+ct^2)\simeq\begin{bmatrix}0&0&0\\1&0&0\\0&1/2&0\\0&0&1/3\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}0\\a\\b/2\\c/3\end{bmatrix}\simeq at+\frac{1}{2}bt^2+\frac{1}{3}ct^3$$

EX 6.3.5: Find standard matrix A for linear transformation $L:P_2\to\mathbb{R}$ s.t. $L(p)=\int_0^1 p(t)\,dt$.

Find the images of the **standard basis vectors** for the **domain** of L: (Write polynomials in **ascending** order)

$$\begin{array}{lllllllll} L(\mathbf{e}_1) & = & L(1) & = & \int_0^1 1\,dt & = \Big[t\Big]_{t=0}^{t=1} & \overset{FTC}{=} & (1)-(0) & = 1\\ L(\mathbf{e}_2) & = & L(t) & = & \int_0^1 t\,dt & = \Big[\frac{1}{2}t^2\Big]_{t=0}^{t=1} & \overset{FTC}{=} & \frac{1}{2}(1)^2-\frac{1}{2}(0)^2 & = \frac{1}{2}\\ L(\mathbf{e}_3) & = & L(t^2) & = & \int_0^1 t^2\,dt & = \Big[\frac{1}{3}t^3\Big]_{t=0}^{t=1} & \overset{FTC}{=} & \frac{1}{3}(1)^3-\frac{1}{3}(0)^3 & = \frac{1}{3}\end{array}\implies \boxed{A=\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{3}\end{bmatrix}}$$

$$\therefore\ L(a+bt+ct^2)\simeq\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{3}\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=a+\frac{1}{2}b+\frac{1}{3}c$$