Change of Basis: Coord. Vector, Transition Matrix

Linear Algebra

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Coordinate Vector Relative to a Basis (Definition)

Definition

(Coordinate Vector Relative to a Basis)

Let V be a finite-dimensional vector space.

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be an ordered basis for V.

Let vector $\mathbf{x} \in V$ s.t. $\mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$

Then the **coordinate vector of $\mathbf{x}$ relative to basis $\mathcal{B}$** is

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = (c_1, c_2, \ldots, c_n)^T$$

where $c_1, c_2, \ldots, c_n$ are the **coordinates of $\mathbf{x}$ relative to basis $\mathcal{B}$**.

$[\mathbf{x}]_{\mathcal{B}}$ is also known as the **$\mathcal{B}$-coordinate vector of $\mathbf{x}$**.
$c_1, c_2, \ldots, c_n$ are also known as the **$\mathcal{B}$-coordinates of $\mathbf{x}$**.
The order of the vectors in the basis is critical, hence the term ordered basis.

Standard Basis for common Vector Spaces

Recall the **standard bases** for common vector spaces:

VECTOR SPACE	STANDARD BASIS	DIM.
$\mathbb{R}^2$	$\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \equiv \left\{ \widehat{\mathbf{i}}, \widehat{\mathbf{j}} \right\}$	2
$\mathbb{R}^3$	$\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \equiv \left\{ \widehat{\mathbf{i}}, \widehat{\mathbf{j}}, \widehat{\mathbf{k}} \right\}$	3
$\mathbb{R}^4$	$\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$	4
$\mathbb{R}^5$	$\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$	5

<u>NOTE:</u> For this section, only vector spaces $\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^4, \mathbb{R}^5$ will be considered.

Proposition

(Converting $[\mathbf{x}]_{\mathcal{B}} \to [\mathbf{x}]_{\mathcal{E}}$)

Let V be a finite-dimensional vector space.

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be an ordered basis for V.

Let $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$ be the ordered standard basis for V.

Let $[\mathbf{x}]_{\mathcal{B}} = (c_1, c_2, \ldots, c_n)^T$.

Then $[\mathbf{x}]_{\mathcal{E}} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n \leftarrow$ (*Simplify linear combination*)

WEX 4-7-1: Let $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\} \equiv \{\mathbf{v}_1, \mathbf{v}_2\}$. Find $[\mathbf{x}]_{\mathcal{E}}$ if $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$.

$$[\mathbf{x}]_{\mathcal{B}} = (-5)\mathbf{v}_1 + (3)\mathbf{v}_2 = (-5)\begin{bmatrix} -2 \\ -1 \end{bmatrix} + (3)\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 19 \\ 17 \end{bmatrix}$$

$$= (19)\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (17)\begin{bmatrix} 0 \\ 1 \end{bmatrix} = (19)\mathbf{e}_1 + (17)\mathbf{e}_2 \implies \boxed{[\mathbf{x}]_{\mathcal{E}} = \begin{bmatrix} 19 \\ 17 \end{bmatrix}}$$

Converting $[\mathbf{x}]_{\mathcal{E}} \to [\mathbf{x}]_{\mathcal{B}}$ (Motivation)

Consider the following example:

Let basis $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\} \equiv \{\mathbf{v}_1, \mathbf{v}_2\} \subseteq \mathbb{R}^2$. Find $[\mathbf{x}]_{\mathcal{B}}$ if $[\mathbf{x}]_{\mathcal{E}} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$.

Recall the standard basis for $\mathbb{R}^2$: $\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \equiv \{\mathbf{e}_1, \mathbf{e}_2\} \subseteq \mathbb{R}^2$

$$(\mathbf{x} \text{ in } \mathcal{B}\text{-coordinates}) = \begin{bmatrix} -3 \\ -2 \end{bmatrix} = (\mathbf{x} \text{ in } \mathcal{E}\text{-coordinates})$$

$$\implies [\mathbf{x}]_{\mathcal{B}} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = \begin{bmatrix} -3 \\ -2 \end{bmatrix} = (-3)\mathbf{e}_1 + (-2)\mathbf{e}_2 = [\mathbf{x}]_{\mathcal{E}}$$

$$\implies c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = \begin{bmatrix} -3 \\ -2 \end{bmatrix} \implies \begin{bmatrix} | & | \\ \mathbf{v}_1 & \mathbf{v}_2 \\ | & | \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$\implies \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix} \implies \left[\begin{array}{cc|c} -2 & 3 & -3 \\ -1 & 4 & -2 \end{array}\right] \implies \left[\ \mathcal{B} \mid [\mathbf{x}]_{\mathcal{E}}\ \right]$$

So, to find c_1, c_2, perform Gauss-Jordan on augmented matrix $\left[\ \mathcal{B} \mid [\mathbf{x}]_{\mathcal{E}}\ \right]$.

Converting $[\mathbf{x}]_{\mathcal{E}} \to [\mathbf{x}]_{\mathcal{B}}$ (Procedure)

Proposition

(Converting $[\mathbf{x}]_{\mathcal{E}} \to [\mathbf{x}]_{\mathcal{B}}$)

Let V be a finite-dimensional vector space.

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be an ordered basis for V.

Let $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$ be the ordered standard basis for V.

GIVEN: Vector $\mathbf{x}$ in standard basis coordinates: $[\mathbf{x}]_{\mathcal{E}}$

TASK: Write vector $\mathbf{x}$ in non-std basis $\mathcal{B}$-coordinates: $[\mathbf{x}]_{\mathcal{B}}$

(1) $[\mathcal{B} \mid [\mathbf{x}]_{\mathcal{E}}] \xrightarrow{Gauss-Jordan} [\, I \mid [\mathbf{x}]_{\mathcal{B}}]$

WEX 4-7-2: Let $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\} \equiv \{\mathbf{v}_1, \mathbf{v}_2\}$. Find $[\mathbf{x}]_{\mathcal{B}}$ if $[\mathbf{x}]_{\mathcal{E}} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$.

$$[\, \mathcal{B} \mid [\mathbf{x}]_{\mathcal{E}} \,] = \left[\begin{array}{cc|c} -2 & 3 & -3 \\ -1 & 4 & -2 \end{array} \right] \xrightarrow{Gauss-Jordan} \left[\begin{array}{cc|c} \boxed{1} & 0 & 6/5 \\ 0 & \boxed{1} & -1/5 \end{array} \right]$$

$$\therefore \quad [\mathbf{x}]_{\mathcal{E}} = \begin{bmatrix} -3 \\ -2 \end{bmatrix} = \left(\frac{6}{5}\right)\mathbf{v}_1 + \left(-\frac{1}{5}\right)\mathbf{v}_2 \implies \boxed{[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 6/5 \\ -1/5 \end{bmatrix}}$$

Converting $[\mathbf{x}]_{\mathcal{B}} \to [\mathbf{x}]_{\mathcal{B}'}$ (Procedure)

Proposition

(Converting $[\mathbf{x}]_{\mathcal{B}} \to [\mathbf{x}]_{\mathcal{B}'}$)

Let V be a <u>finite-dimensional</u> vector space.

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be an ordered basis for V.
Let $\mathcal{B}' = \{\mathbf{v}'_1, \mathbf{v}'_2, \ldots, \mathbf{v}'_n\}$ be another ordered basis for V.

Let $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$ be the ordered <u>standard basis</u> for V.

<u>GIVEN:</u> Vector $\mathbf{x}$ in non-std basis $\mathcal{B}$-coordinates: $[\mathbf{x}]_{\mathcal{B}}$

<u>TASK:</u> Write vector $\mathbf{x}$ in non-std basis $\mathcal{B}'$-coordinates: $[\mathbf{x}]_{\mathcal{B}'}$

(1) *Convert* $[\mathbf{x}]_{\mathcal{B}} \to [\mathbf{x}]_{\mathcal{E}}$ *(2)* $[\mathcal{B}' \mid [\mathbf{x}]_{\mathcal{E}}] \xrightarrow{Gauss-Jordan} [\, I \mid [\mathbf{x}]_{\mathcal{B}'}]$

Converting $[\mathbf{x}]_{\mathcal{B}} \to [\mathbf{x}]_{\mathcal{B}'}$ (Example)

WEX 4-7-3: Let $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ and $\mathcal{B}' = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$.

Find $[\mathbf{x}]_{\mathcal{B}'}$ if $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

Recall the standard basis for $\mathbb{R}^2$: $\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \equiv \{\mathbf{e}_1, \mathbf{e}_2\} \subseteq \mathbb{R}^2$

1^{st}, convert $[\mathbf{x}]_{\mathcal{B}} \to [\mathbf{x}]_{\mathcal{E}}$: $[\mathbf{x}]_{\mathcal{B}} = (5)\begin{bmatrix} -2 \\ -1 \end{bmatrix} + (3)\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix} = [\mathbf{x}]_{\mathcal{E}}$

$$[\mathcal{B}' \mid [\mathbf{x}]_{\mathcal{E}}] = \left[\begin{array}{cc|c} -1 & 2 & -1 \\ 2 & 3 & 7 \end{array}\right] \xrightarrow{Gauss-Jordan} \left[\begin{array}{cc|c} \boxed{1} & 0 & 17/7 \\ 0 & \boxed{1} & 5/7 \end{array}\right]$$

$$\therefore \; [\mathbf{x}]_{\mathcal{E}} = \begin{bmatrix} -1 \\ 7 \end{bmatrix} = \left(\frac{17}{7}\right)\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \left(\frac{5}{7}\right)\begin{bmatrix} 2 \\ 3 \end{bmatrix} \Longrightarrow \boxed{[\mathbf{x}]_{\mathcal{B}'} = \begin{bmatrix} 17/7 \\ 5/7 \end{bmatrix}}$$

Transition Matrix (Definition)

Definition

(Transition Matrix)

Let $\mathcal{B}, \mathcal{B}'$ be two ordered bases for finite-dimensional vector space V.
Let vector $\mathbf{x} \in V$. Then:

The **transition matrix** $\underset{\mathcal{B}' \leftarrow \mathcal{B}}{P}$ **from** $\mathcal{B}$ to $\mathcal{B}'$ satisfies $\underset{\mathcal{B}' \leftarrow \mathcal{B}}{P} [\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{B}'}$

The **transition matrix** $\underset{\mathcal{B} \leftarrow \mathcal{B}'}{P}$ **from** $\mathcal{B}'$ to $\mathcal{B}$ satisfies $\underset{\mathcal{B} \leftarrow \mathcal{B}'}{P} [\mathbf{x}]_{\mathcal{B}'} = [\mathbf{x}]_{\mathcal{B}}$

Corollary

(Transition Matrices are Square)

Let $\mathcal{B}, \mathcal{B}'$ be two ordered bases for finite-dimensional vector space V.

Then transition matrices $\underset{\mathcal{B}' \leftarrow \mathcal{B}}{P}$ and $\underset{\mathcal{B} \leftarrow \mathcal{B}'}{P}$ are square matrices.

REMARK: **Never** use the book's notation for transition matrix: P or P^{-1}

Inverse of a Transition Matrix

The inverse of a transition matrix is precisely what one would expect:

Theorem

(Inverse of a Transition Matrix)

Let $\mathcal{B}, \mathcal{B}'$ be two ordered bases for finite-dimensional vector space V.

Then $\left(\underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}\right)^{-1} = \underset{\mathcal{B}'\leftarrow\mathcal{B}}{P}$ *and* $\left(\underset{\mathcal{B}'\leftarrow\mathcal{B}}{P}\right)^{-1} = \underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}$

PROOF: $[\mathbf{x}]_{\mathcal{B}} = \underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}[\mathbf{x}]_{\mathcal{B}'} = \underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}\left(\underset{\mathcal{B}'\leftarrow\mathcal{B}}{P}[\mathbf{x}]_{\mathcal{B}}\right)$

$$\implies [\mathbf{x}]_{\mathcal{B}} = \left(\underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}\right)\left(\underset{\mathcal{B}'\leftarrow\mathcal{B}}{P}\right)[\mathbf{x}]_{\mathcal{B}}$$

$$\implies \left(\underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}\right)\left(\underset{\mathcal{B}'\leftarrow\mathcal{B}}{P}\right) = I \qquad \text{[Since } \mathbf{v} = A\mathbf{v} \ \forall \mathbf{v} \implies A = I\text{]}$$

$$\implies \left(\underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}\right)^{-1} = \underset{\mathcal{B}'\leftarrow\mathcal{B}}{P} \quad \text{and} \quad \left(\underset{\mathcal{B}'\leftarrow\mathcal{B}}{P}\right)^{-1} = \underset{\mathcal{B}\leftarrow\mathcal{B}'}{P}$$

QED

Finding the Transition Matrix (Procedure)

So how to systematically find a transition matrix between bases??

Theorem

(Finding the Transition Matrix)

GIVEN: Ordered bases $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ *and* $\mathcal{B}' = \{\mathbf{v}'_1, \mathbf{v}'_2, \ldots, \mathbf{v}'_n\}$ *for* $\mathbb{R}^n$.

TASK: Find the transition matrix $\underset{\mathcal{B}' \leftarrow \mathcal{B}}{P}$ *from* $\mathcal{B}$ *to* $\mathcal{B}'$.

(1) $[\mathcal{B}' \mid \mathcal{B}] \xrightarrow{Gauss-Jordan} \left[I \;\middle|\; \underset{\mathcal{B}' \leftarrow \mathcal{B}}{P} \right]$

Finding the Transition Matrix (Example)

<u>WEX 4-7-4:</u> Let $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ and $\mathcal{B}' = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$.

(a) Find the transition matrix $\underset{\mathcal{B}' \leftarrow \mathcal{B}}{P}$.

(b) Find $[\mathbf{x}]_{\mathcal{B}'}$ if $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

(c) Find the transition matrix $\underset{\mathcal{B} \leftarrow \mathcal{B}'}{P}$.

(d) Find $[\mathbf{x}]_{\mathcal{B}}$ if $[\mathbf{x}]_{\mathcal{B}'} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

Finding the Transition Matrix (Example)

WEX 4-7-4: Let $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ and $\mathcal{B}' = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$.

(a) Find the transition matrix $\underset{\mathcal{B}' \leftarrow \mathcal{B}}{P}$. (b) Find $[\mathbf{x}]_{\mathcal{B}'}$ if $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

(c) Find the transition matrix $\underset{\mathcal{B} \leftarrow \mathcal{B}'}{P}$. (d) Find $[\mathbf{x}]_{\mathcal{B}}$ if $[\mathbf{x}]_{\mathcal{B}'} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

(a) Apply Gauss-Jordan to augmented matrix $[\mathcal{B}' \mid \mathcal{B}]$:

$$[\mathcal{B}' \mid \mathcal{B}] = \left[\begin{array}{cc|cc} -1 & 2 & -2 & 3 \\ 2 & 3 & -1 & 4 \end{array} \right] \xrightarrow{Gauss-Jordan} \left[\begin{array}{cc|cc} \boxed{1} & 0 & \frac{4}{7} & -\frac{1}{7} \\ 0 & \boxed{1} & -\frac{5}{7} & \frac{10}{7} \end{array} \right] = \left[I \;\middle|\; \underset{\mathcal{B}' \leftarrow \mathcal{B}}{P} \right]$$

(b) $[\mathbf{x}]_{\mathcal{B}'} = \underset{\mathcal{B}' \leftarrow \mathcal{B}}{P} [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 4/7 & -1/7 \\ -5/7 & 10/7 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \boxed{\begin{bmatrix} 17/7 \\ 5/7 \end{bmatrix}}$

Finding the Transition Matrix (Example)

WEX 4-7-4: Let $\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ and $\mathcal{B}' = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$.

(a) Find the transition matrix $\underset{\mathcal{B}' \leftarrow \mathcal{B}}{P}$.

(b) Find $[\mathbf{x}]_{\mathcal{B}'}$ if $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.

(c) Find the transition matrix $\underset{\mathcal{B} \leftarrow \mathcal{B}'}{P}$.

(d) Find $[\mathbf{x}]_{\mathcal{B}}$ if $[\mathbf{x}]_{\mathcal{B}'} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

(c) Apply Gauss-Jordan to augmented matrix $[\mathcal{B} \mid \mathcal{B}']$:

$$[\mathcal{B} \mid \mathcal{B}'] = \left[\begin{array}{cc|cc} -2 & 3 & -1 & 2 \\ -1 & 4 & 2 & 3 \end{array}\right] \xrightarrow{Gauss-Jordan} \left[\begin{array}{cc|cc} \boxed{1} & 0 & 2 & \frac{1}{5} \\ 0 & \boxed{1} & 1 & \frac{4}{5} \end{array}\right] = \left[I \;\middle|\; \underset{\mathcal{B} \leftarrow \mathcal{B}'}{P} \right]$$

(d) $[\mathbf{x}]_{\mathcal{B}} = \underset{\mathcal{B} \leftarrow \mathcal{B}'}{P} [\mathbf{x}]_{\mathcal{B}'} = \begin{bmatrix} 2 & 1/5 \\ 1 & 4/5 \end{bmatrix} \begin{bmatrix} -2 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} -3 \\ 2 \end{bmatrix}}$

Fin

Fin.