VECTOR FUNCTIONS: ALGEBRA & LIMITS [SST 10.1]

EX 10.1.7: Let $\mathbf{F}(t) = \langle 1, t, t^2 \rangle$ and $\mathbf{G}(t) = \left(e^t\right)\widehat{\mathbf{i}} - (\cos t)\widehat{\mathbf{k}}$.

Compute: (a) $3\mathbf{F}(t) - 2t^2\mathbf{G}(t)$ (b) $\mathbf{F}(t) \cdot \mathbf{G}(t)$ (c) $\mathbf{F}(t) \times \mathbf{G}(t)$ (d) $\mathbf{G}(t) \times \mathbf{F}(t)$

First of all, write vector function $\vec{\mathbf{G}}(t)$ in **component form**: $\vec{\mathbf{G}}(t) = \langle e^t, 0, -\cos t \rangle$

Perform the prescribed vector operations as usual, simplifying whenever possible:

(a) $3\vec{\mathbf{F}}(t) - 2t^2\vec{\mathbf{G}}(t) = 3\left\langle 1, t, t^2 \right\rangle - \left(2t^2\right)\left\langle e^t, 0, -\cos t \right\rangle = \left\langle 3, 3t, 3t^2 \right\rangle - \left\langle 2t^2 e^t, 0, -2t^2 \cos t \right\rangle = \boxed{\left\langle 3 - 2t^2 e^t, 3t, 3t^2 + 2t^2 \cos t \right\rangle}$

(b) $\vec{\mathbf{F}}(t) \cdot \vec{\mathbf{G}}(t) = \left\langle 1, t, t^2 \right\rangle \cdot \left\langle e^t, 0, -\cos t \right\rangle = (1)\left(e^t\right) + (t)(0) + \left(t^2\right)(-\cos t) = \boxed{e^t - t^2 \cos t}$

(c) $$\vec{\mathbf{F}}(t) \times \vec{\mathbf{G}}(t) = \left\langle 1, t, t^2 \right\rangle \times \left\langle e^t, 0, -\cos t \right\rangle = \begin{vmatrix} \widehat{\mathbf{i}} & \widehat{\mathbf{j}} & \widehat{\mathbf{k}} \\ 1 & t & t^2 \\ e^t & 0 & -\cos t \end{vmatrix} = \begin{vmatrix} t & t^2 \\ 0 & -\cos t \end{vmatrix} \widehat{\mathbf{i}} - \begin{vmatrix} 1 & t^2 \\ e^t & -\cos t \end{vmatrix} \widehat{\mathbf{j}} + \begin{vmatrix} 1 & t \\ e^t & 0 \end{vmatrix} \widehat{\mathbf{k}}$$

$$= (-t\cos t - 0)\widehat{\mathbf{i}} - \left(-\cos t - t^2 e^t\right)\widehat{\mathbf{j}} + \left(0 - te^t\right)\widehat{\mathbf{k}} = \boxed{\left\langle -t\cos t, \cos t + t^2 e^t, -te^t \right\rangle}$$

(d) $\vec{\mathbf{G}}(t) \times \vec{\mathbf{F}}(t) = -\left[\vec{\mathbf{F}}(t) \times \vec{\mathbf{G}}(t)\right] = \boxed{\left\langle t\cos t, -\cos t - t^2 e^t, te^t \right\rangle}$

As used in part (d), special properties of vector operations carry over to the corresponding operations on vector functions.