EX 11.2.15: Let $h(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2} & \text{, for } (x,y) \neq (0,0) \\ k & \text{, for } (x,y) = (0,0) \end{cases}$, where $k \in \mathbb{R}$.

What value of k ensures that h is continuous at $(0,0)$?

Recall the requirements for h to be continuous at $(0,0)$:

- $h(0,0)$ must exist
- $\lim\limits_{(x,y)\to(0,0)} h(x,y)$ must exist
- $\lim\limits_{(x,y)\to(0,0)} h(x,y) = h(0,0)$

$h(0,0) = k$ for some $k \in \mathbb{R}$, so clearly $h(0,0)$ exists.

Hence, the desired value of k is precisely the value of $\lim\limits_{(x,y)\to(0,0)} h(x,y)$:

$$\lim_{(x,y)\to(0,0)} h(x,y) = \lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} \overset{NS}{=} \frac{(0)^3+(0)^3}{(0)^2+(0)^2} = \frac{0}{0} \implies \text{Rewrite/Simplify}$$

$$\overset{POLAR}{=} \lim_{r\to 0^+} \frac{(r\cos\theta)^3+(r\sin\theta)^3}{(r\cos\theta)^2+(r\sin\theta)^2} = \lim_{r\to 0^+} \frac{r^3(\cos^3\theta+\sin^3\theta)}{r^2(\cos^2\theta+\sin^2\theta)} = \lim_{r\to 0^+} \frac{r(\cos^3\theta+\sin^3\theta)}{1} \overset{NS}{=} (0)(\cos^3\theta+\sin^3\theta) = 0$$

$\therefore$ $\boxed{h \text{ is continuous at } (0,0) \text{ precisely when } k = 0}$