

**EX 11.3.10:** Let  $f(x, y, z) = x^2y^3z^4$ . Compute:

$$(a) \quad \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left[ \frac{\partial f}{\partial x} \right] = \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial x} [x^2y^3z^4] \right] = \frac{\partial}{\partial x} [2xy^3z^4] = \boxed{2y^3z^4}$$

$$(b) \quad \frac{\partial^2 f}{\partial z \partial x} = \frac{\partial}{\partial z} \left[ \frac{\partial f}{\partial x} \right] = \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial x} [x^2y^3z^4] \right] = \frac{\partial}{\partial z} [2xy^3z^4] = \boxed{8xy^3z^3}$$

$$(c) \quad \frac{\partial^3 f}{\partial z^3} = \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial z} \left[ \frac{\partial f}{\partial z} \right] \right] = \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial z} [x^2y^3z^4] \right] \right] = \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial z} [4x^2y^3z^3] \right] = \frac{\partial}{\partial z} [12x^2y^3z^2] = \boxed{24x^2y^3z}$$

$$(d) \quad \frac{\partial^3 f}{\partial y^2 \partial x} = \frac{\partial}{\partial y} \left[ \frac{\partial}{\partial y} \left[ \frac{\partial f}{\partial x} \right] \right] = \frac{\partial}{\partial y} \left[ \frac{\partial}{\partial y} \left[ \frac{\partial}{\partial x} [x^2y^3z^4] \right] \right] = \frac{\partial}{\partial y} \left[ \frac{\partial}{\partial y} [2xy^3z^4] \right] = \frac{\partial}{\partial y} [6xy^2z^4] = \boxed{12xyz^4}$$

$$(e) \quad \frac{\partial^3 f}{\partial x \partial z^2} = \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial z} \left[ \frac{\partial f}{\partial z} \right] \right] = \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial z} [x^2y^3z^4] \right] \right] = \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial z} [4x^2y^3z^3] \right] = \frac{\partial}{\partial x} [12x^2y^3z^2] = \boxed{24xy^3z^2}$$

$$(f) \quad \frac{\partial^3 f}{\partial x \partial y \partial z} = \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial y} \left[ \frac{\partial f}{\partial z} \right] \right] = \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial y} \left[ \frac{\partial}{\partial z} [x^2y^3z^4] \right] \right] = \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial y} [4x^2y^3z^3] \right] = \frac{\partial}{\partial x} [12x^2y^2z^3] = \boxed{24xy^2z^3}$$