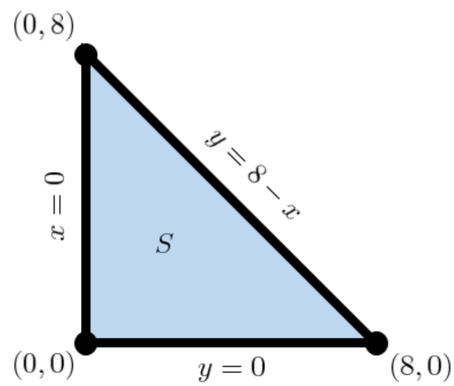
EX 11.7.6: Let $h(x,y) = 2 + 2x + 2y - x^2 - y^2$ and set S be the closed triangle with vertices $(0,0)$, $(0,8)$, $(8,0)$. Find the extreme values of h over S and the points at which they occur.

STEP 1: Find all CP's of h

$$\begin{cases} h_x = \frac{\partial}{\partial x}\left[2 + 2x + 2y - x^2 - y^2\right] = 2 - 2x \overset{set}{=} 0 \\ h_y = \frac{\partial}{\partial y}\left[2 + 2x + 2y - x^2 - y^2\right] = 2 - 2y \overset{set}{=} 0 \end{cases} \implies \begin{cases} x = 1 \\ y = 1 \end{cases} \implies \text{Only CP of } h \text{ is } (1,1)$$

STEP 2: Sketch set S and label all BC's & BP's



STEP 3: Discard any CP's not in S

CP $(1,1)$ is in S, so no CP's to discard.

STEP 4: Find all BCP's

Along BC $x = 0$: Let $h_1(y) = h(0,y) = 2 + 2(0) + 2y - (0)^2 - y^2 = 2 + 2y - y^2$

$\implies h_1'(y) = \frac{d}{dy}\left[2 + 2y - y^2\right] = 2 - 2y \overset{set}{=} 0 \implies y = 1 \implies (0,1)$ is a BCP

Along BC $y = 0$: Let $h_2(x) = h(x,0) = 2 + 2x + 2(0) - x^2 - (0)^2 = 2 + 2x - x^2$

$\implies h_2'(x) = \frac{d}{dx}\left[2 + 2x - x^2\right] = 2 - 2x \overset{set}{=} 0 \implies x = 1 \implies (1,0)$ is a BCP

Along BC $y = 8 - x$: Let $h_3(x) = h(x, 8-x) = 2 + 2x + 2(8-x) - x^2 - (8-x)^2 = -46 + 16x - 2x^2$

$\implies h_3'(x) = \frac{d}{dx}\left[-46 + 16x - 2x^2\right] = 16 - 4x \overset{set}{=} 0 \implies x = 4 \implies y = 8 - 4 = 4 \implies (4,4)$ is a BCP

STEP 5: Build table computing h at each BP, BCP, and undiscarded CP

(x,y)	**(1,1)**	$(0,0)$	**(0,8)**	**(8,0)**	$(0,1)$	$(1,0)$	$(4,4)$
$h(x,y)$	**4**	2	**-46**	**-46**	3	3	-14
Result	**Abs Max**		**Abs Min**	**Abs Min**			
Type	**CP**	BP	**BP**	**BP**	BCP	BCP	BCP