**EX 12.2.5:** **Setup** double integral(s) for the volume of the solid $E$ below surface $z = 6 - 2x^2 - 3y^2$ and above plane $z = 0$.

For clarity, let $f(x, y) = 6 - 2x^2 - 3y^2$

In order to determine region $D$, intersect the surface $z = f(x, y)$ with the plane $z = 0$ by equating their RHS's:

$$6 - 2x^2 - 3y^2 = 0 \implies 2x^2 + 3y^2 = 6 \implies \frac{x^2}{3} + \frac{y^2}{2} = 1 \implies \frac{x^2}{(\sqrt{3})^2} + \frac{y^2}{(\sqrt{2})^2} = 1 \quad \leftarrow \text{Ellipse with axial radii } \sqrt{3}, \sqrt{2}$$

Hence, the ellipse $\frac{x^2}{3} + \frac{y^2}{2} = 1$ is the only BC of region $D$.

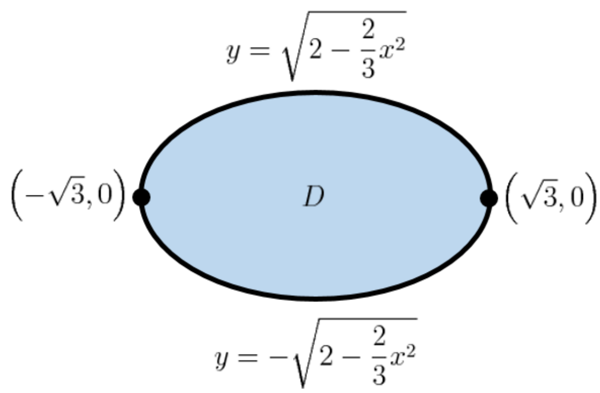
Notice that $f(x, y) > 0$ **inside** this ellipse: [e.g. $f(0,0) = 6 - 2(0)^2 - 3(0)^2 = 6 > 0$]

Thus, region $D$ (which is the bottom face of solid $E$) is the **closed elliptic disk**: $\frac{x^2}{3} + \frac{y^2}{2} \le 1$

## SUPPOSE THE INTEGRATION ORDER $dy\ dx$ IS DESIRED:

Solve the elliptic BC for $y$: $\frac{x^2}{3} + \frac{y^2}{2} = 1 \implies y = \pm\sqrt{2 - \frac{2}{3}x^2}$

Label the two BP's whose $y$-coord is zero, meaning they separate the ellipse into two BC's:

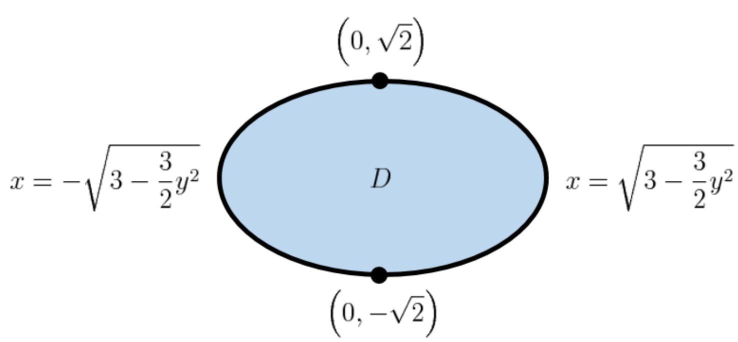



Therefore, $\boxed{\text{Volume}(E) = \iint_D f\ dA = \int_{\text{Smallest } x\text{-coord}}^{\text{Largest } x\text{-coord}} \int_{\text{Btm BC}}^{\text{Top BC}} f(x,y)\ dy\ dx = \int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{2-\frac{2}{3}x^2}}^{\sqrt{2-\frac{2}{3}x^2}} \left(6 - 2x^2 - 3y^2\right) dy\ dx}$

## SUPPOSE THE INTEGRATION ORDER $dx\ dy$ IS DESIRED:

Solve the elliptic BC for $x$: $\frac{x^2}{3} + \frac{y^2}{2} = 1 \implies x = \pm\sqrt{3 - \frac{3}{2}y^2}$

Label the two BP's whose $x$-coord is zero, meaning they separate the ellipse into two BC's:




Therefore, $\boxed{\text{Volume}(E) = \iint_D f\ dA = \int_{\text{Smallest } y\text{-coord}}^{\text{Largest } y\text{-coord}} \int_{\text{Left BC}}^{\text{Right BC}} f(x,y)\ dx\ dy = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{3-\frac{3}{2}y^2}}^{\sqrt{3-\frac{3}{2}y^2}} \left(6 - 2x^2 - 3y^2\right) dx\ dy}$

**EX 12.2.6:** **Setup** double integral(s) for the volume of solid $E$ bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $z = 7 - 3x - 2y$.

Trivially, the intersection of the $yz$-plane ($x = 0$) & $xy$-plane ($z = 0$) is the $y$-axis ($x = 0$).

Trivially, the intersection of the $xz$-plane ($y = 0$) & $xy$-plane ($z = 0$) is the $x$-axis ($y = 0$).

Let $f(x, y) = 7 - 3x - 2y$ and intersect the plane $z = f(x, y)$ with the $xy$-plane ($z = 0$):

$f(x, y) = 0 \implies 7 - 3x - 2y = 0 \implies y = \frac{7}{2} - \frac{3}{2}x$ $\leftarrow$ slanted line with $y$-intercept $\frac{7}{2}$

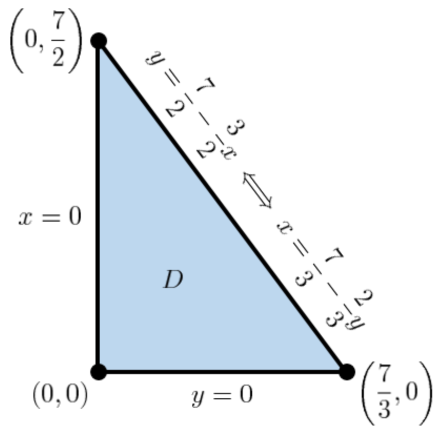
Hence, the lines $x = 0$, $y = 0$, and $y = \frac{7}{2} - \frac{3}{2}x$ are the three BC's of region $D$.

Moreover, the three lines form a **triangle**.

Notice that $f(x, y) > 0$ **inside** this triangle: [e.g. $f(1, 1) = 7 - 3(1) - 2(1) = 2 > 0$]

Thus, region $D$ (which is the bottom face of solid $E$) is the resulting **closed triangle**.

Sketch region $D$




Notice that region $D$ is both V-Simple & H-Simple, meaning either integration order will yield only one double integral.

**SUPPOSE THE INTEGRATION ORDER $dy\ dx$ IS DESIRED:**

Then, $\text{Volume}(E) = \iint_D f\ dA = \int_{\text{Smallest } x\text{-coord}}^{\text{Largest } x\text{-coord}} \int_{\text{Btm BC}}^{\text{Top BC}} f(x, y)\ dy\ dx = \int_0^{\frac{7}{3}} \int_0^{\frac{7}{2} - \frac{3}{2}x} (7 - 3x - 2y)\ dy\ dx$

**SUPPOSE THE INTEGRATION ORDER $dx\ dy$ IS DESIRED:**

Then, $\text{Volume}(E) = \iint_D f\ dA = \int_{\text{Smallest } y\text{-coord}}^{\text{Largest } y\text{-coord}} \int_{\text{Left BC}}^{\text{Right BC}} f(x, y)\ dx\ dy = \int_0^{\frac{7}{2}} \int_0^{\frac{7}{3} - \frac{2}{3}y} (7 - 3x - 2y)\ dx\ dy$