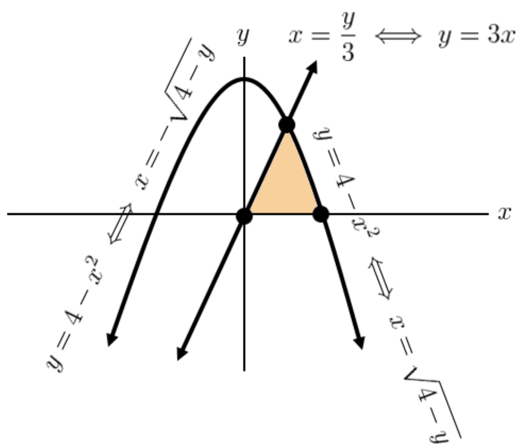
EX 12.2.8: Let $I = \int_0^3 \int_{y/3}^{\sqrt{4-y}} f(x,y)\ dx\ dy$.

Sketch the region of integration & write an equivalent iterated double integral with the order of integration reversed.

Since the given order is $dx\ dy$, the right BC is $x = y/3 \iff y = 3x$ & the left BC is $x = \sqrt{4-y} \iff y = 4 - x^2$.

Moreover, the smallest y-coord in the region is 0 & the largest y-coord in the region is 3.

Using this info, sketch the BC's of the region of integration & shade the region itself:



Now, find the three BP's of region D:

Intersect line $x = \frac{y}{3}$ with x-axis $y = 0$: $y = 0 \implies x = \frac{(0)}{3} = 0 \implies$ BP is $(0,0)$

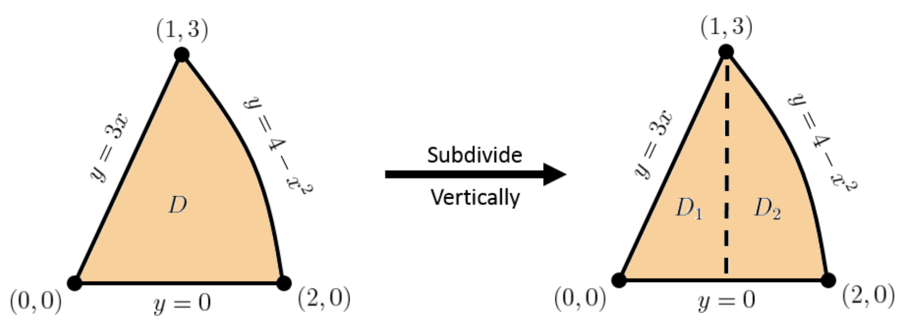
Intersect parabola $x = \sqrt{4-y}$ with x-axis $y = 0$: $y = 0 \implies x = \sqrt{4-(0)} = \sqrt{4} = 2 \implies$ BP is $(2,0)$

Intersect parabola $y = 4 - x^2$ with line $y = 3x$:

$4 - x^2 = 3x \implies x^2 + 3x - 4 = 0 \implies (x+4)(x-1) = 0 \implies x = 1, x = -4$

Since region D is in Quadrant I $(x, y > 0)$, discard $x = -4$. So, $x = 1 \implies y = 3(1) = 3 \implies$ BP is $(1,3)$

Re-sketch region D larger in size and without the clutter, writing BC's only in terms of x since the $dy\ dx$ order is desired:



Notice that region D is not V-simple, so subdivide vertically along BP $(1,3)$ into two subregions D_1 & D_2.

$$\iint_{D_1} f\ dA = \int_{\text{Smallest } x\text{-coord in } D_1}^{\text{Largest } x\text{-coord in } D_1} \int_{\text{Btm BC in } D_1}^{\text{Top BC in } D_1} f(x,y)\ dy\ dx = \int_0^1 \int_0^{3x} f(x,y)\ dy\ dx$$

$$\iint_{D_2} f\ dA = \int_{\text{Smallest } x\text{-coord in } D_2}^{\text{Largest } x\text{-coord in } D_2} \int_{\text{Btm BC in } D_2}^{\text{Top BC in } D_2} f(x,y)\ dy\ dx = \int_1^2 \int_0^{4-x^2} f(x,y)\ dy\ dx$$

Therefore, $I = \iint_D f\ dA = \iint_{D_1} f\ dA + \iint_{D_2} f\ dA = \boxed{\int_0^1 \int_0^{3x} f(x,y)\ dy\ dx + \int_1^2 \int_0^{4-x^2} f(x,y)\ dy\ dx}$