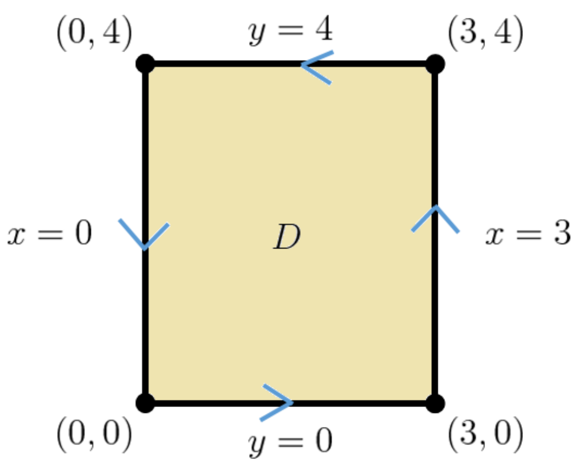
EX 13.4.2: Let Γ be the positively oriented rectangle with vertices $(0,0)$, $(3,0)$, $(0,4)$, $(3,4)$.

Use Green's Theorem to compute $I = \oint_\Gamma \left[2xy\ dx + x\cos(\pi y)\ dy\right]$.

Let $\vec{\mathbf{F}}(x,y) = \langle M(x,y), N(x,y)\rangle$, where $M(x,y) = 2xy$ and $N(x,y) = x\cos(\pi y)$

Compute the cross partials: $\dfrac{\partial M}{\partial y} = \dfrac{\partial}{\partial y}\left[2xy\right] = 2x$ $\quad \dfrac{\partial N}{\partial x} = \dfrac{\partial}{\partial x}\left[x\cos(\pi y)\right] = \cos(\pi y)$

Since Green's Thm results in a double integral, **sketch path Γ & region D enclosed by Γ, labeling BC's & BP's**:



Observe that region D is **simply connected** & path Γ is a **Jordan curve**, so Green's Theorem is applicable.

Moreover, region D is both V-Simple & H-Simple, so only one iterated double integral's needed for order $dy\,dx$ or $dx\,dy$.

$$
\begin{aligned}
I &= \oint_\Gamma \left[M\ dx + N\ dy\right] \\
&\overset{GREEN}{=} \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA \\
&= \iint_D \left[\cos(\pi y) - 2x\right]\ dA \\
&= \int_{\text{Smallest } y\text{-coord}}^{\text{Largest } y\text{-coord}} \int_{\text{Left BC of } D}^{\text{Right BC of } D} \left[\cos(\pi y) - 2x\right]\ dx\ dy \qquad \text{(Picking the order } dy\ dx \text{ also works fine)} \\
&= \int_0^4 \int_0^3 \left[\cos(\pi y) - 2x\right]\ dx\ dy \\
&= \int_0^4 \left[x\cos(\pi y) - x^2\right]_{x=0}^{x=3} dy \\
&\overset{FTC}{=} \int_0^4 \left[3\cos(\pi y) - 9\right]\ dy \\
&= \left[\frac{3}{\pi}\sin(\pi y) - 9y\right]_{y=0}^{y=4} \\
&\overset{FTC}{=} \frac{3}{\pi}\sin(4\pi) - 9(4) \\
&= \boxed{-36}
\end{aligned}
$$