DOUBLE INTEGRALS: RECTANGULAR COORDINATES [SST 12.2]

- **PROPERTIES OF DOUBLE INTEGRALS:**
  - Let set D be a closed & bounded region in $\mathbb{R}^2$.
    Let functions $f(x,y)$ & $g(x,y)$ be defined on D.
    Let $k \in \mathbb{R}$.

    Constant Multiple Rule: $\iint_D kf\ dA = k\iint_D f\ dA$

    Sum/Difference Rule: $\iint_D \left[f \pm g\right]\ dA = \iint_D f\ dA \pm \iint_D g\ dA$

    Nonnegativity Rule: $f(x,y) \geq 0\ \ \forall (x,y) \in D \implies \iint_D f\ dA \geq 0$

    Dominance Rule: $f(x,y) \leq g(x,y)\ \ \forall (x,y) \in D \implies \iint_D f\ dA \leq \iint_D g\ dA$

    Region Additivity Rule: $D = D_1 \cup D_2 \implies \iint_D f\ dA = \iint_{D_1} f\ dA + \iint_{D_2} f\ dA$

- **AREA OF A V-SIMPLE REGION D:**
  - $\text{Area}(D) := \iint_D\ dA = \int_{\text{smallest } x\text{-value in } D}^{\text{largest } x\text{-value in } D} \int_{\text{bottom BC of } D}^{\text{top BC of } D} dy\ dx = \int_a^b \int_{g_1(x)}^{g_2(x)} dy\ dx$

- **AREA OF A H-SIMPLE REGION D:**
  - $\text{Area}(D) := \iint_D\ dA = \int_{\text{smallest } y\text{-value in } D}^{\text{largest } y\text{-value in } D} \int_{\text{left BC of } D}^{\text{right BC of } D} dx\ dy = \int_c^d \int_{h_1(y)}^{h_2(y)} dx\ dy$

- **AREA OF A REGION THAT'S NEITHER V-SIMPLE NOR H-SIMPLE:**
  - Subdivide region into subregions each of which are V-Simple or H-Simple.
  - Use the Region Additivity Rule for Double Integrals to find the area.

- **VOLUME OF SOLID E BOUNDED BELOW BY xy-PLANE & BOUNDED ABOVE BY SURFACE $z = f(x,y)$:**
  - $\text{Volume}(E) = \iint_D f\ dA$ (Use above to write as iterated integrals)

- **SETTING UP A DOUBLE INTEGRAL TO FIND AREA OR VOLUME:**
  - Sketch region D and label all BC's & BP's.
  - Determine whether D is V-Simple, H-Simple, Both, or Neither.
    - * If D is neither V-Simple nor H-Simple, subdivide region.
  - Write appropriate iterated double integral(s). (see above)

- **INTERCHANING THE ORDER OF INTEGRATION OF A DOUBLE INTEGRAL:**
  - Sketch the **region of integration** (label it D) and label all BC's & BP's.
  - Determine whether D is V-Simple, H-Simple, Both, or Neither.
    - * If D is neither V-Simple nor H-Simple, subdivide region.
  - Write appropriate iterated double integral(s) using the reversed order of integration. (see above)

EX 12.2.1: Compute $I = \int_0^{\pi/2} \int_0^{\sin x} e^y \cos x \; dy \; dx$.

EX 12.2.2: Compute $I = \iint_D (2x + 3y)\ dA$, where region $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 2\}$

EX 12.2.3: Compute $I = \iint_D \frac{\sin x}{x}\, dA$, where D is the triangle bounded by lines $y = 0$, $x = \pi$, and $y = x$.

EX 12.2.4: **Setup** double integral(s) for the area of the region D bounded by curves $x = 2 - y^2$ and $y = -x$.

EX 12.2.5: **Setup** double integral(s) for the volume of the solid E below surface $z = 6 - 2x^2 - 3y^2$ and above plane $z = 0$.

<u>EX 12.2.6:</u> **Setup** double integral(s) for the volume of solid E bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $z = 7 - 3x - 2y$.

EX 12.2.7: Let $I = \int_0^1 \int_x^{2-x} f(x,y)\, dy\, dx$.

Sketch the region of integration & write an equivalent iterated double integral with the order of integration reversed.

EX 12.2.8: Let $I = \int_0^3 \int_{y/3}^{\sqrt{4-y}} f(x,y)\ dx\ dy$.

Sketch the region of integration & write an equivalent iterated double integral with the order of integration reversed.

EX 12.2.9: Compute $I = \int_0^{\pi/4} \int_0^{\pi/2} x\cos(2x+y)\ dy\ dx$.