GREEN'S THEOREM [SST 13.4]

- **CLOSED CURVES:**

  A **closed curve** is a curve that begins & ends at the same point.

  Special notation is used with line integrals along **closed curves**:

$$\oint_C f\ ds \qquad \oint_C f\ dx \qquad \oint_C f\ dy \qquad \oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{R}}$$

- **JORDAN CURVES:**

  A **Jordan curve** is a piecewise smooth closed curve that does not intersect itself.

- **POSITIVELY ORIENTED JORDAN CURVES:**

  A Jordan curve is **positively oriented** if when moving along the curve via a parameterization, the interior region of the curve stays to your left as parameter t increases.

- **GREEN'S THEOREM FOR SIMPLY-CONNECTED REGIONS:**

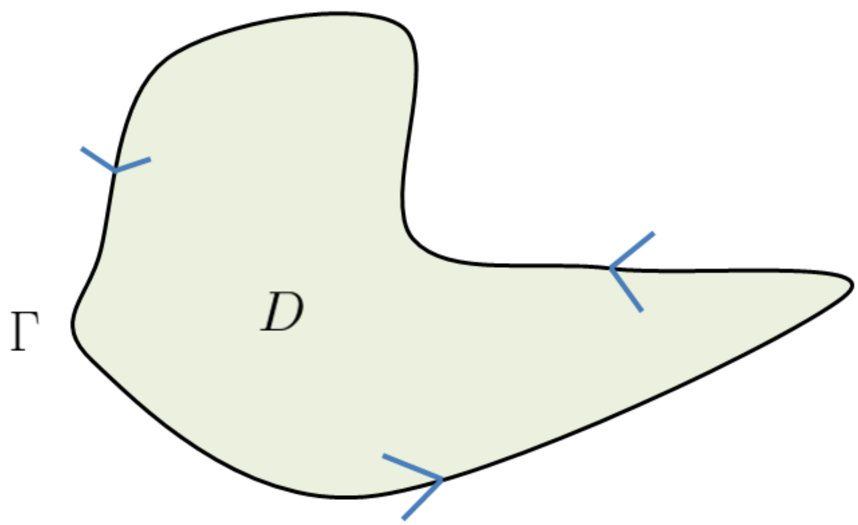
  Let $D \subset \mathbb{R}^2$ be a simply-connected region in the xy-plane. (see picture below)

  Let Γ be a positively oriented piecewise smooth Jordan curve that bounds region D. (see picture below)

  Let vector field $\vec{\mathbf{F}} \in C^{(1,1)}(D)$ s.t. $\vec{\mathbf{F}}(x,y) = \langle M(x,y), N(x,y) \rangle$.

  Then

$$\oint_\Gamma (M\ dx + N\ dy) = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)\ dA$$



EX 13.4.1: Let C be the positively oriented quarter-circle with boundary points $(0,0)$, $(2,0)$, $(0,2)$.

Use Green's Theorem to compute $I = \oint_C \Big[(5x - 6y + 7)\ dx + (3x + 2y + 1)\ dy\Big]$.

EX 13.4.2: Let Γ be the positively oriented rectangle with vertices $(0,0)$, $(3,0)$, $(0,4)$, $(3,4)$.

Use Green's Theorem to compute $I = \oint_{\Gamma} \left[2xy\ dx + x\cos(\pi y)\ dy\right]$.

EX 13.4.3: Let C be the positively oriented trapezoid with vertices $(0,0)$, $(3,0)$, $(0,4)$, $(2,4)$.

Use Green's Theorem to compute $I = \oint_C \left[\left(\arctan\left(\sqrt{x}\right) + y^3\right)\, dx - \left(3x^2 - \sin\left(y^2\right)\right)\, dy \right]$.