STOKES' THEOREM [SST 13.6]

- **COMPATIBLE ORIENTATIONS:**

  The orientations of a surface S & its bounding Jordan curve C are said to be **compatible** if the positive direction on C is counterclockwise in relation to the outward normal vector $\widehat{\mathbf{N}}$ of the surface (see below figure).

- **STOKES' THEOREM:**

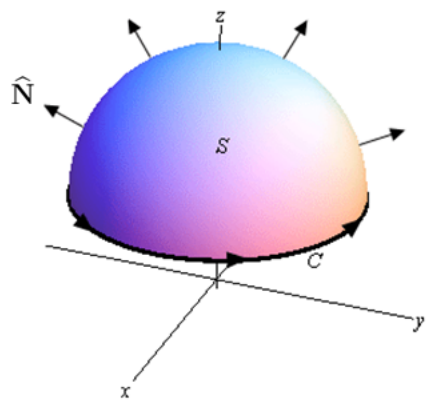
  Let orientable surface $S \subset \mathbb{R}^3$ have the unit normal vector field $\widehat{\mathbf{N}}$.

  Moreover, let S be bounded by a Jordan curve C whose orientation is compatible with the orientation on S.

  Let vector field $\vec{\mathbf{F}} \in C^{(1,1,1)}(S)$.

  Then:

$$\oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{R}} = \iint_S \left(\nabla \times \vec{\mathbf{F}}\right) \cdot \widehat{\mathbf{N}}\, dS$$



- **STOKES' THEOREM (FTC FORM):**

  Let orientable surface $S \subset \mathbb{R}^3$ have the unit normal vector field $\widehat{\mathbf{N}}$.

  Moreover, let the boundary of S, denoted by ∂S, be a Jordan curve C whose orientation is compatible with that of S.

  Let vector field $\vec{\mathbf{F}} \in C^{(1,1,1)}(S)$.

  Then:

$$\iint_S \left(\nabla \times \vec{\mathbf{F}}\right) \cdot \widehat{\mathbf{N}}\, dS = \oint_{\partial S} \vec{\mathbf{F}} \cdot d\vec{\mathbf{R}}$$

- **CHOOSING A SURFACE FOR A BOUNDARY JORDAN CURVE:**

JORDAN BOUNDARY CURVE	**CANDIDATE SURFACE(S)**
Circle	Paraboloid, Half-Sphere, Circular Cone, or Plane
Ellipse	Paraboloid, Half-Ellipsoid, Elliptic Cone, or Plane
Polygon	Plane
Intersection of Surfaces S_1 and S_2	S_1 or S_2

EX 13.6.1: Let curve C be the circle $x^2 + y^2 = 1$ in the plane $z = 1$ oriented CCW as viewed from above.

Use Stokes' Theorem to compute the line integral $I = \oint_C (y\ dx + x\ dy + z\ dz)$.

EX 13.6.2: Let curve C be the intersection of plane $y+z=2$ & cylinder $x^2+y^2=1$ oriented CCW as viewed from above.

Use Stokes' Theorem to compute the line integral $I = \oint_C \left(-y^2\ dx + x\ dy + z^2\ dz\right)$.

EX 13.6.3: Let surface S be the portion of the cone $z = -\sqrt{x^2+y^2}$ with $z \geq -2$ with upward unit normal $\widehat{\mathbf{N}}$.

Let vector field $\vec{\mathbf{F}}(x,y,z) = \langle yz, -xz, z^3 \rangle$.

Use Stokes' Theorem to compute the surface integral $I = \iint_S \left(\nabla \times \vec{\mathbf{F}}\right) \cdot \widehat{\mathbf{N}}\, dS$.