DIVERGENCE THEOREM (AKA GAUSS' THEOREM) [SST 13.7]

- **GAUSS' THEOREM:**

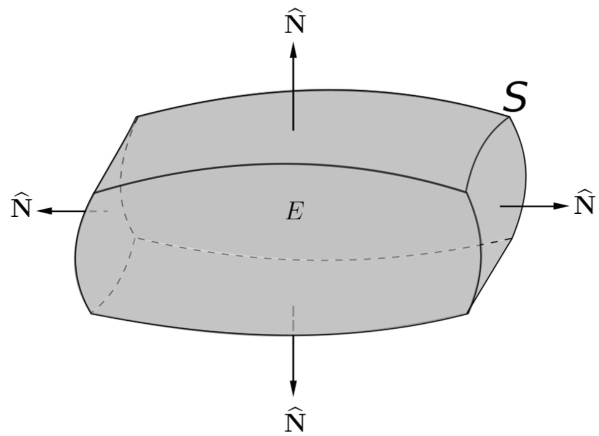
  Let closed piecewise smooth surface $S \subset \mathbb{R}^3$ be oriented outward with the unit normal field $\widehat{\mathbf{N}}$.

  Let simply-connected solid $E \subset \mathbb{R}^3$ be the interior of surface S, i.e. S is the outer boundary surface of solid E.

  Let vector field $\vec{\mathbf{F}} \in C^{(1,1,1)}(E)$.

  Then:

$$\oiint_S \vec{\mathbf{F}} \cdot \widehat{\mathbf{N}}\, dS = \iiint_E \nabla \cdot \vec{\mathbf{F}}\, dV$$



- **GAUSS' THEOREM (FTC FORM):**

  Let solid $E \subset \mathbb{R}^3$ be simply-connected.

  Let the boundary of E, denoted by ∂E, be a closed piecewise smooth surface oriented outward with unit normal $\widehat{\mathbf{N}}$.

  Let vector field $\vec{\mathbf{F}} \in C^{(1,1,1)}(E)$.

  Then:

$$\iiint_E \nabla \cdot \vec{\mathbf{F}}\, dV = \oiint_{\partial E} \vec{\mathbf{F}} \cdot \widehat{\mathbf{N}}\, dS$$

EX 13.7.1: Let surface S be the sphere $x^2 + y^2 + z^2 = 4$ oriented outward with unit normal field $\widehat{\mathbf{N}}$.

Let vector field $\vec{\mathbf{F}}(x, y, z) = \langle x + 1, 2y + z^4, 3z - \cos(xy) \rangle$.

Use the Divergence Theorem (AKA Gauss' Theorem) to compute the surface integral $I = \oiint_S \vec{\mathbf{F}} \cdot \widehat{\mathbf{N}}\, dS$.

EX 13.7.2: Let solid E be bounded by the planes $z = 0$, $y = 0$, $y = 2$, and the parabolic cylinder $z = 1 - x^2$.

Let surface S be the boundary of solid E with outward unit normal $\widehat{\mathbf{N}}$.

Let vector field $\vec{\mathbf{F}}(x, y, z) = \langle x + \arctan y, 2y + e^z, z^2 - \sin x \rangle$.

Use the Divergence Theorem (AKA Gauss' Theorem) to compute the surface integral $I = \oiint_S \vec{\mathbf{F}} \cdot \widehat{\mathbf{N}} \, dS$.

EX 13.7.3: Let surface S_1 be the hemisphere $y = -\sqrt{2 - x^2 - z^2}$ and surface S_2 be the disk $\left\{ \begin{array}{c} x^2 + z^2 = 2 \\ y = 0 \end{array} \right\}$ on xz-plane.

Let surface $S = S_1 \cup S_2$ with outward unit normal $\widehat{\mathbf{N}}$ and let vector field $\vec{\mathbf{F}}(x, y, z) = \langle x^3, y, 3 \rangle$.

Use the Divergence Theorem (AKA Gauss' Theorem) to compute the surface integral $I = \oiint_S \vec{\mathbf{F}} \cdot \widehat{\mathbf{N}}\, dS$.