EX 11.6.3: Let $f(x,y) = \ln\sqrt{x^2+y^2}$.

(a) In what direction is f increasing most rapidly from point $P(1,2)$?

First, use properties of logarithms to make differentiation easier: $f(x,y) = \ln\sqrt{x^2+y^2} = \ln\left(x^2+y^2\right)^{1/2} = \frac{1}{2}\ln\left(x^2+y^2\right)$

$$\nabla f = \left\langle \frac{1}{2}\cdot\frac{2x}{x^2+y^2}, \frac{1}{2}\cdot\frac{2y}{x^2+y^2}\right\rangle = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right\rangle \implies \nabla f(1,2) = \left\langle \frac{1}{1^2+2^2}, \frac{2}{1^2+2^2}\right\rangle = \boxed{\left\langle \frac{1}{5}, \frac{2}{5}\right\rangle}$$

(b) What is the maximum rate of increase of f from point $P(1,2)$?

$$||\nabla f(1,2)|| = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \sqrt{\frac{5}{25}} = \boxed{\frac{1}{\sqrt{5}}}$$

(c) In what direction is f decreasing most rapidly from point $P(1,2)$?

$$-\nabla f(1,2) = \boxed{\left\langle -\frac{1}{5}, -\frac{2}{5}\right\rangle}$$

EX 11.6.4: Let $h(x,y,z) = \tan(x+2y+3z)$.

(a) In what direction is h increasing most rapidly from point $P(-5,1,1)$?

$$\nabla h = \langle h_x, h_y, h_z\rangle = \left\langle \sec^2(x+2y+3z), 2\sec^2(x+2y+3z), 3\sec^2(x+2y+3z)\right\rangle$$

$$\nabla h(-5,1,1) = \left\langle \sec^2((-5)+2(1)+3(1)), 2\sec^2((-5)+2(1)+3(1)), 3\sec^2((-5)+2(1)+3(1))\right\rangle = \boxed{\langle 1,2,3\rangle}$$

(b) What is the maximum rate of increase of h from point $P(-5,1,1)$?

$$||\nabla h(-5,1,1)|| = ||\langle 1,2,3\rangle|| = \sqrt{1^2+2^2+3^2} = \boxed{\sqrt{14}}$$

(c) In what direction is h decreasing most rapidly from point $P(-5,1,1)$?

$$-\nabla h(-5,1,1) = \boxed{\langle -1,-2,-3\rangle}$$

EX 11.6.5: Find a unit vector that's normal to the hyperbola $\frac{x^2}{5} - \frac{y^2}{3} = 1$ at point $Q(\sqrt{10}, \sqrt{3})$.

The key here is to realize that the desired normal unit vector is $\widehat{\mathbf{n}} = \nabla f(\sqrt{10},\sqrt{3})$ or $\widehat{\mathbf{n}} = -\nabla f(\sqrt{10},\sqrt{3})$

Let $f(x,y) = $ (Left-Hand Side of Canonical Form of Hyperbola) $= \frac{x^2}{5} - \frac{y^2}{3}$

Then, $\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right\rangle = \left\langle \frac{2}{5}x, -\frac{2}{3}y\right\rangle \implies \nabla f(\sqrt{10},\sqrt{3}) = \left\langle \frac{2}{5}(\sqrt{10}), -\frac{2}{3}(\sqrt{3})\right\rangle = \left\langle \frac{4}{\sqrt{10}}, -\frac{2}{\sqrt{3}}\right\rangle$

Hence, $\mathbf{n} = \left\langle \frac{4}{\sqrt{10}}, -\frac{2}{\sqrt{3}}\right\rangle$

Finally, **normalize n**: $\widehat{\mathbf{n}} = \frac{\mathbf{n}}{||\mathbf{n}||} = \frac{\left\langle \frac{4}{\sqrt{10}}, -\frac{2}{\sqrt{3}}\right\rangle}{\sqrt{\left(\frac{4}{\sqrt{10}}\right)^2 + \left(-\frac{2}{\sqrt{3}}\right)^2}} = \frac{\sqrt{30}}{\sqrt{88}}\left\langle \frac{4}{\sqrt{10}}, -\frac{2}{\sqrt{3}}\right\rangle = \boxed{\left\langle \frac{2\sqrt{3}}{\sqrt{22}}, -\frac{\sqrt{10}}{\sqrt{22}}\right\rangle}$