Double Integrals: Surface Area

Calculus III

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TTU

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Arc Length of Portion of Curves $y = f(x)$ & $x = g(y)$

Proposition

(Arc Length)

Let $f(x) \in C^1[a,b]$.
Then the ***arc length*** *of* $f(x)$ *over interval* $[a,b]$ *is*

$$\int_a^b ds := \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Proposition

(Arc Length)

Let $g(y) \in C^1[c,d]$.
Then the ***arc length*** *of* $g(y)$ *over interval* $[c,d]$ *is*

$$\int_c^d ds := \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$$

REMARK: $dx, dy \equiv$ length elements $\qquad ds \equiv$ arc length element

Surface Area of Portion of Surface $z = f(x, y)$

Proposition

(Surface Area)

Let $f(x,y) \in C^{(1,1)}(D)$ *where* D *is a region in the* xy*-plane.*
Let S *be the portion of the surface* $z = f(x,y)$ *that lies directly over* D*.*
(Region D *is also known as the* ***projection*** *of* S *onto the* xy*-plane.)*

Then the ***surface area*** *of* S *is*

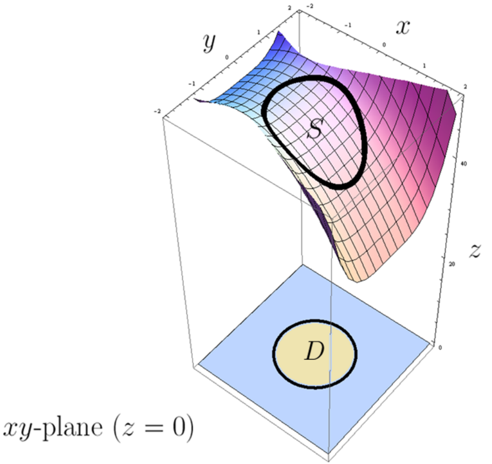
$$\iint_S dS = \iint_D \sqrt{1 + [f_x]^2 + [f_y]^2}\, dA = \iint_D \sqrt{1 + \left[\frac{\partial f}{\partial x}\right]^2 + \left[\frac{\partial f}{\partial y}\right]^2}\, dA$$

REMARK: $dA \equiv$ area element $\qquad dS \equiv$ surface area element

REMARK: If the resulting double integral is tedious to compute, consider **rewriting integral in polar coordinates**.

PROOF: See the textbook – it's lengthy.

$$f(x, y) = 50 - 2(x+1)^2 + y^4$$



$$\text{Surface Area}(S) := \iint_S dS = \iint_D \sqrt{1 + [f_x]^2 + [f_y]^2}\, dA$$

Surface Area of Portion of Surface $y = g(x, z)$ (Optional)

Proposition

(Surface Area)

Let $g(x, z) \in C^{(1,1)}(D^{[xz]})$ *where* $D^{[xz]}$ *is a region in the* xz*-plane.*
Let S *be the portion of the surface* $y = g(x, z)$ *that lies directly over* $D^{[xz]}$*.*
(Region $D^{[xz]}$ *is also known as the* ***projection*** *of* S *onto the* xz*-plane.)*

Then the ***surface area*** *of* S *is*

$$\iint_S dS = \iint_{D^{[xz]}} \sqrt{1 + [g_x]^2 + [g_z]^2}\, dA = \iint_{D^{[xz]}} \sqrt{1 + \left[\frac{\partial g}{\partial x}\right]^2 + \left[\frac{\partial g}{\partial z}\right]^2}\, dA$$

REMARK: $dA \equiv$ area element $\qquad$ $dS \equiv$ surface area element

REMARK: If the resulting double integral is tedious to compute, consider **rewriting integral in polar coordinates**.

Surface Area of Portion of Surface $x = h(y, z)$ (Optional)

Proposition

(Surface Area)

Let $h(y,z) \in C^{(1,1)}(D^{[yz]})$ *where* $D^{[yz]}$ *is a region in the* yz*-plane.*
Let S *be the portion of the surface* $x = h(y,z)$ *that lies directly over* $D^{[yz]}$*.*
(Region $D^{[yz]}$ *is also known as the* ***projection*** *of* S *onto the* yz*-plane.)*

Then the ***surface area*** *of* S *is*

$$\iint_S dS = \iint_{D^{[yz]}} \sqrt{1 + [h_y]^2 + [h_z]^2}\, dA = \iint_{D^{[yz]}} \sqrt{1 + \left[\frac{\partial h}{\partial y}\right]^2 + \left[\frac{\partial h}{\partial z}\right]^2}\, dA$$

REMARK: $dA \equiv$ area element $dS \equiv$ surface area element

REMARK: If the resulting double integral is tedious to compute, consider **rewriting integral in polar coordinates**.

Fin.