Section 1.1

- Why do we care - important tool to model growth and changes
- Derivatives - rate of change
- $F(x, y, y', \dots y^n) = 0$
- Differen types : ODE, PDE
- ODE - "simple", only involving 1 Variable
- PDE - "complex", involving several Variables

Ex: ODE = $y'' - \cos x\, y' + y = 3x$

PDE = $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial z}$

- order - highest derivative, tells you how many solutions might be there
  - ↳ $1^{st} = y' - 2xy - y^2 = 0$
  - ↳ $2nd = y'' - \cos x\, y' + y = 3x$
- if $y' = 0$ then $y = c$ is a solution

  if $y'' = 0$ then $y = \underbrace{x, 0, 2x, 3x, 5x+2}_{\text{Special Solutions}} = \underbrace{ax+b}_{\text{general Solution}}$ = any constant solution
- Linearity: an n^{th} order ODE is linear if it is of the form as follows:

  ↳ $a_n(x)\, y^{(n)}(x) + a_{n-1}(x)\, y^{(n-1)}(x) + \dots\, a(x) y' + a_0(x) y = g(x)$

Ex: Linear = $y'' - \cos x y' + y = 3x$

non Linear = $(y')^2 = x^2 y^2$ because $y' \cdot y'$ = multiplication between 2 derivatives

- Homogeneity: $g(x) = 0$

  Ex: $y' - 2xy - y^2 = 0$
- non homogeneity : does not equal 0, $g(x) \neq 0$

  Ex: $y'' - \cos x\, y' + y = 3x$
- Solution : Varifies the equation

  Ex: $y' - 3y = 0 \rightarrow y = e^{3x} \rightarrow y' = 3e^{3x} = 3y \rightarrow y' - 3y = 0$ so $y = e^{3x}$ is a solution

Section 1.2 - inital value problems

- IVP : solving $F(x, y, \dots, y^{(n)}) = 0$ (2 steps)

Subject to / Constraints
$$\begin{cases} y(x_0) = y_1 \\ y'(x_0) = y_2 \\ \vdots \\ y^{(n-1)}(x_0) = y_n \end{cases}$$

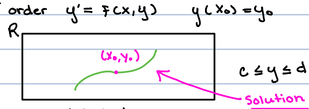
- Constraints = n^{th} order

Ex: $y' = 3x^2$ (D.E.), $y(2) = -1$ (IVP)

1. general solution : $y = x^3 + c$, $y' = 3x^2 + 0$
2. Find constants next: $y(2) = 2^3 + c = -1 \rightarrow c = -9$, so $y = x^3 - 9$ is a solution to IVP.

- Fund. Existence and uniqueness Theorem

  Consider 1^{st} order $y' = f(x, y)$ $y(x_0) = y_0$



- Hypothesis: $f(x,y)$ and $\frac{\partial f}{\partial y}$ are cont. on R (rectangle)
- Conclusion : There is a solution!

Ex: $y' = -y^2$, $y(0) = 1$, $f(x,y) = -y^2$, $f_y = -2y$ are contin in R^2 then ⇒ IVP has a unique solution

- Theorem for 1^{st} order linear IVP.

  Normal Form : $a_1(x) y' + a_0(x) y = g(x)$, $y(x_0) = y_0$

  $\Longleftrightarrow y' = \frac{g(x) - a_0(x)\, y}{a_2(x)} = f(x, y)$

- Hypothesis: $g(x)$, $a_0(x)$, $a_2(x)$ are cont. on some interval I

  $a_1(x) \neq 0$ on I
- Conclusion : There is a solution on I

Ex: $(x^2-9)y' + x\cos x\, y = \frac{x+1}{x}$, $y(1) = 5$ $\leftarrow$ Linear

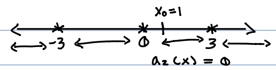
$a_1(x) = x^2-9$, $a_0(x) = x\cos x$, $g(x) = \frac{x+1}{x}$

$a_1(x)$, $a_0(x)$ are cont. everywhere

$g(x)$ is cont as long as $x \neq 0$

$a_1(x) = 0$ at $x \pm 3$

Real line



Maximal interval = largest interval containing x_0 = $(0, 3)$, so that is a unique solution to IVP $(0, 3)$