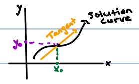
Section 2.1

- Solution Curves

$y' = f(x,y)$



- Slope of the tangent line is exactly $y' = f(x,y)$
- Doing it systematically – Slope field used to find solution

– Autonomous Equations : $y' = f(y)$

- Goal – to understand behavior of solutions without actually finding those solutions
- Critical points – Aka : Nodes, Equalibrium points, fixed points

  represent solutions that are constant

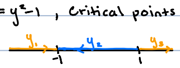
Ex: $y' = \overbrace{y^2 - y}^{f(y)}$, $f(y) = y^2 - 1 = 0 \longleftrightarrow y = \pm 1$

$y' = 0,\ y^2 - 1 = 1^2 - 1 = 0$

function: $y = 1$ is a solution

- This leads to the classification of critical points :
  1. Asymptohically stable (Sink): if solutions begin near y_0 approach y_0 as x increases, moving
  2. Unstable (source): if solution that begin near y_0, move away from y_0 as x increases
  3. Semi-stable: neither a sink or source, stable on one side and unstable on another side

Ex: $y' = y^2 - 1$, critical points : $y = \pm 1$



$y_1 < -1,\ y' = y^2 - 1$ at y_1 $y_1^2 - 1 > 0$, y_1 increase towards -1

$-1 < y_2 < 1,\ y' = y^2 - 1$ at y_2, $y_2^2 - 1 < 0$, y decreases towards -1

-1: stable

1: not stable → source

$y_3 > 1,\ y' = y^2 - 1$ at y_3, $y_3^2 - 1 > 0$, y_3 increases away from 1

Summary:

1. set function $= 0$, Solve, Find critical points
2. draw line, plot critical points
3. check sign of $f(y)$ in each interval
   ↳ $f(y) > 0,\ y\uparrow$, increase
   $f(y) < 0,\ y\downarrow$, decrease
4. mark arrows on interval and make classification
   ↳ arrows point to number, $\longrightarrow \# \longleftarrow$, ie; increase, decrease = Stable
   arrows point away from number, $\longleftarrow \# \longrightarrow$, ie; decrease, increase = unstable
   arrows point same direction , $\longrightarrow \# \longrightarrow$ or $\longleftarrow \# \longleftarrow$, ie; increase, increase or decrease, decrease = Semi-stable

Section 2.2

- Separable Equations – 1[st] order, ODE of the form, keyword = seperable

$\frac{dy}{dx} = g(x)h(y)$

Solving: Seperate x away from y

$\frac{dy}{dx} = g(x)h(y) \longleftrightarrow \frac{dy}{h(y)} = g(x)\,dx$ integrate both sides

$\int \frac{dy}{h(y)} = \int g(x)\,dx + c$, Evaluate indefinite integrals → Implicit solutions

Ex: $\frac{dy}{dx} = \frac{y}{1+x} = y \cdot \frac{1}{x}$

1. $\frac{dy}{y} = \frac{dx}{1+x}$
2. $\int \frac{dy}{y} = \int \frac{dx}{1+x}$
3. $\ln(y) + c = \ln(1+x) + c$ = implicit general solution

Don't need c on both sides

Implicit: relation of y and x

Explicit: y written as a function of x

4. $e^{\ln(y)} = e^{\ln(1+x)+c}$
5. $y = e^c(1+x)$ = explicit solution

- Remember – opposite of derivative is Integral

Ex: $y' = (x+1)\cos(x^2+2x)$

$\frac{dy}{dx} = (x+1)\cos(x^2+2x)$

$\int dy = \int (x+1)\cos(x^2+2x)\,dx$

$y = \frac{1}{2}\sin(x^2+2x) + c$

u-sub: $u = x^2+2x$, $du = 2x+2\,dx$, $\int (x+1)\cos(x^2+2x)\,dx = \int \cos u \frac{du}{2} = \frac{1}{2}\sin u + c$, $\frac{1}{2}\sin(x^2+2x)$

Ex: $y' = (x+1)\cos(x^2+2x)$, $y(0) = 2020$ (IVP)

1. Find general solutional
2. plug in numbers (inital conditions) to determine constants

$y = \frac{1}{2}\sin(x^2+2x) + c$

$y(0) = 2020 \rightarrow 2020 = \frac{1}{2}\sin(0^2+2\cdot 0) + c$

$2020 = \frac{1}{2}\sin(0) + c = c$

Solution to IVP: $y = \frac{1}{2}\sin(x^2+2x) + 2020$