A (logic) statement is a declarative sentence that is either true or false.

Examples of Statements:
- "Janice works the morning shift."
- "I prefer cats to gerbils."
- "You expect the next iPad to be released within three years."
- Mathematical Statements: \[1 + 1 = 2, \quad 4 - 5 = 7, \quad 4 \neq 5, \quad 9 \neq 9, \quad \ldots\]

The following are not statements:
- Questions: "How much profit was made last quarter?"
- Commands: "Mow the front lawn."
- Exclamations: "Alright!", "Hey!", \ldots
- Onomatopoeia: "Tic, Toc!", "Wham!", "Chirp!", "Woof!", \ldots
- Paradoxes: "This sentence is false."
**Definition**

(Simple Statement)

A simple statement contains a single idea.

**Definition**

(Compound Statement)

A compound statement contains several simple statements (ideas). The ideas in a compounded statement are "connected" by connectives. Moreover, the ideas can be represented by variables: \( P, Q, R, \ldots \)

<table>
<thead>
<tr>
<th>CONNECTIVE NAME:</th>
<th>NOTATION:</th>
<th>MEANING:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conjunction</td>
<td>( P \land Q )</td>
<td>( P ) and ( Q )</td>
</tr>
<tr>
<td>Disjunction</td>
<td>( P \lor Q )</td>
<td>( P ) or ( Q )</td>
</tr>
<tr>
<td>Negation</td>
<td>( \sim P )</td>
<td>not ( P )</td>
</tr>
<tr>
<td>Conditional</td>
<td>( P \rightarrow Q )</td>
<td>if ( P ) then ( Q )</td>
</tr>
<tr>
<td>Biconditional</td>
<td>( P \leftrightarrow Q )</td>
<td>( P ) if and only if ( Q )</td>
</tr>
</tbody>
</table>

REMARK: Then symbol \( \equiv \) means "represents"
WEX 3-1-1: Let $P \equiv \text{"Roses are red"}$, $Q \equiv \text{"Violets are blue"}$. Express each symbolic statement in English:

(a) $P \land Q$
(b) $\sim P \lor \sim Q$
(c) $\sim P \rightarrow Q$
(d) $P \leftrightarrow \sim Q$
WEX 3-1-1: Let $P \equiv "Roses are red"$, $Q \equiv "Violets are blue"$. Express each symbolic statement in English:

(a) $P \land Q$ \hspace{2cm} "Roses are red and violets are blue"
(b) $\sim P \lor \sim Q$
(c) $\sim P \rightarrow Q$
(d) $P \leftrightarrow \sim Q$
WEX 3-1-1: Let $P \equiv \text{"Roses are red"}$, $Q \equiv \text{"Violets are blue"}$. Express each symbolic statement in English:

(a) $P \land Q$ \quad \text{"Roses are red and violets are blue"} \\
(b) $\sim P \lor \sim Q$ \quad \text{"Roses are not red or violets are not blue"} \\
(c) $\sim P \rightarrow Q$ \\
(d) $P \leftrightarrow \sim Q$
WEX 3-1-1: Let \( P \equiv \) "Roses are red", \( Q \equiv \) "Violets are blue". Express each symbolic statement in English:

(a) \( P \land Q \) "Roses are red and violets are blue"

(b) \( \sim P \lor \sim Q \) "Roses are not red or violets are not blue"

(c) \( \sim P \rightarrow Q \) "If roses are not red, then violets are blue"

(d) \( P \iff \sim Q \)
Compound Statement & Connectives

WEX 3-1-1: Let \( P \equiv "\text{Roses are red}" \), \( Q \equiv "\text{Violets are blue}" \). Express each symbolic statement in English:

(a) \( P \land Q \) "Roses are red and violets are blue"

(b) \( \sim P \lor \sim Q \) "Roses are not red or violets are not blue"

(c) \( \sim P \rightarrow Q \) "If roses are not red, then violets are blue"

(d) \( P \leftrightarrow \sim Q \) "Roses are red if and only if violets are not blue"
Quantifiers & Quantified Statements (Definition)

**Definition**

(Quantifiers)

**Quantifiers** express **how many** "objects" satisfy a given property or idea. A **quantified statement** is a statement with **at least one quantifier**.

**Universal Quantifiers**: "All", "Every", "Each"

**Existential Quantifiers**: "Some", "At least one", "There exists", "There is/are"

**WARNING**: Quantifier "Any" can be either universal or existential!

Examples of **quantified statements**:

- "All roses are red", "Every rose is red", "Each rose is red"
- "Some violets are blue", "At least one violet is blue", "There exists a blue violet", "There is a blue violet", "There are blue violets"
Sometimes, the **negation** of a **quantified statement** must be considered:

<table>
<thead>
<tr>
<th>QUANTIFIED STATEMENT</th>
<th>NEGATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Some....are....&quot;</td>
<td>&quot;No....are....&quot;</td>
</tr>
<tr>
<td>&quot;All....are....&quot;</td>
<td>&quot;Some....are not....&quot;</td>
</tr>
<tr>
<td>&quot;No....are....&quot;</td>
<td>&quot;Some....are....&quot;</td>
</tr>
<tr>
<td>&quot;Some....are not....&quot;</td>
<td>&quot;All....are....&quot;</td>
</tr>
</tbody>
</table>
WEX 3-1-2: Negate the quantified statements:

(a) "Some roses are red"
(b) "All violets are blue"
(c) "No violets are blue"
(d) "Some roses are not red"
WEX 3-1-2: Negate the quantified statements:

(a) "Some roses are red" \[\text{"No roses are red"}\]

(b) "All violets are blue"

(c) "No violets are blue"

(d) "Some roses are not red"
WEX 3-1-2: Negate the quantified statements:

(a) "Some roses are red" → "No roses are red"
(b) "All violets are blue" → "Some violets are not blue"
(c) "No violets are blue"
(d) "Some roses are not red"
WEX 3-1-2: Negate the quantified statements:

(a) "Some roses are red"  "No roses are red"
(b) "All violets are blue"  "Some violets are not blue"
(c) "No violets are blue"  "Some violets are blue"
(d) "Some roses are not red"
WEX 3-1-2: Negate the quantified statements:

(a) "Some roses are red"  "No roses are red"

(b) "All violets are blue"  "Some violets are not blue"

(c) "No violets are blue"  "Some violets are blue"

(d) "Some roses are not red"  "All roses are red"
Fin.