CHAPTER 5

The Production Process and Costs
Chapter Outline

• The production function
  – Short-versus long-run decisions
  – Measures of productivity
  – Manager’s role in production process
  – Algebraic forms of the production function and productivity
  – Isoquants and isocosts
  – Cost minimization and optimal input substitution

• The cost function
  – Short-run costs
  – Average and marginal costs
  – Relations among costs
  – Fixed and sunk costs
  – Algebraic forms of cost functions
  – Long-run costs and economies of scale

• Multiple-output cost functions
  – Economies of scope and cost complementarity
Introduction

• Chapter 4 focused on how consumers adjust consumption decisions in reaction to price and income changes. The theory developed illustrates the underlying principles of individual and market demand curves.

• This chapter examines how managers select the optimal mix of inputs that minimize production costs.
The Production Function

• Mathematical function that defines the maximum amount of output that can be produced with a given set of inputs.

\[ Q = F(K, L) \]

, where

– \( Q \) is the level of output.
– \( K \) is the quantity of capital input.
– \( L \) is the quantity of labor input.
Short-Run versus Long-Run Decisions: Fixed and Variable Inputs

• Short-run
  – Period of time where some factors of production (inputs) are *fixed*, and constrain a manager’s decisions.

• Long-run
  – Period of time over which all factors of production (inputs) are *variable*, and can be adjusted by a manager.
Measures of Productivity

• Total product (TP)
  – Maximum level of output that can be produced with a given amount of inputs.

• Average product (AP)
  – A measure of the output produced per unit of input.
    • Average product of labor: \( AP_L = \frac{Q}{L} \)
    • Average product of capital: \( AP_K = \frac{Q}{K} \)

• Marginal product (MP)
  – The change in total product (output) attributable to the last unit of an input.
    • Marginal product of labor: \( MP_L = \frac{\Delta Q}{\Delta L} \)
    • Marginal product of capital: \( MP_K = \frac{\Delta Q}{\Delta K} \)
Measures of Productivity in Action

• Consider the following production function when 5 units of labor and 10 units of capital are combined produce: \( Q = F(10,5) = 150 \).

• Compute the average product of labor.

\[
AP_L = \frac{150}{5} = 30 \text{ units per worker}
\]

• Compute the average product of capital.

\[
AP_L = \frac{150}{10} = 15 \text{ units capital unit}
\]
Relation between Productivity Measures in Action

- Total product
- Average product
- Marginal product

The Production Function

Increasing marginal returns to labor
Decreasing marginal returns to labor
Negative marginal returns to labor

Total product \((TP)\)
Average product \((AP_L)\)
Marginal product \((MP_L)\)
Labor input (holding capital constant)
The Manager’s Role in the Production Process

• Produce output on the production function.
  – Aligning incentives to induce maximum worker effort.

• Use the right mix of inputs to maximize profits.
  – To maximize profits when labor or capital vary in the short run, the manager will hire:

  • Labor until the value of the marginal product of labor equals the wage rate: \( VMP_L = w \), where \( VMP_L = P \times MP_L \)
  • Capital until the value of the marginal product of capital equals the rental rate: \( VMP_K = r \), where \( VMP_K = P \times MP_K \)
Manager’s Role in the Production Process in Action

• Suppose a firm sells its output in a competitive market where its output is sold at $5 per unit. If workers are also hired at a competitive wage of $200, what is the marginal productivity of the last worker?

• Since, $VMP_L = 5 \times MP_L$ and $w = 200$, then,

\[ 5 \times MP_L = 200 \Rightarrow MP_L = 40 \text{ units} \]

– The marginal productivity of the last unit of labor is 40 units.

– Alternatively, management should hire labor such that the last unit of labor produces 40 units.
Algebraic Forms of Production Functions

• Commonly used algebraic production function forms:
  
  – Linear: $Q = F(K, L) = aK + bL$, where $a$ and $b$ are constants.

  – Leontief: $Q = F(K, L) = \min\{aK, bL\}$, where $a$ and $b$ are constants.

  – Cobb-Douglas: $Q = F(K, L) = K^a L^b$, where $a$ and $b$ are constants.
Algebraic Forms of Production Functions in Action

• Suppose that a firm’s estimated production function is:

\[ Q = 3K + 6L \]

• How much output is produced when 3 units of capital and 7 units of labor are employed?

\[ Q = F(3,7) = 3(3) + 6(7) = 51 \text{ units} \]
Algebraic Measures of Productivity

• Given the commonly used algebraic production function forms, we can compute the measures of productivity as follows:
  – Linear:
    • Marginal products: \( MP_K = a \) and \( MP_L = b \)
    • Average products: \( AP_K = \frac{aK + bL}{K} \) and \( AP_L = \frac{aK + bL}{L} \)
  – Cobb-Douglas:
    • Marginal products: \( MP_K = aK^{a-1}L^b \) and \( MP_L = bK^{a-1}L^b \)
    • Average products: \( AP_K = \frac{K^aL^b}{K} \) and \( AP_L = \frac{K^aL^b}{L} \)
Algebraic Measures of Productivity in Action

• Suppose that a firm produces output according to the production function
\[ Q = F(1, L) = (1)^{1/4}L^{3/4} \]

• Which is the fixed input?
  – Capital is the fixed input.

• What is the marginal product of labor when 16 units of labor is hired?
\[ MP_L = 1 \times \frac{3}{4}L^{-\frac{1}{4}} = 1 \times \frac{3}{4}(16)^{-\frac{1}{4}} = \frac{3}{8} \]
Isoquants and Marginal Rate of Technical Substitution

• *Isoquants* capture the tradeoff between combinations of inputs that yield the same output in the long run, when all inputs are variable.

• Marginal rate of technical substitutions (MRTS)
  – The rate at which a producer can substitute between two inputs and maintain the same level of output.
  – Absolute value of the slope of the isoquant.

\[
MRTS_{KL} = \frac{MP_L}{MP_K}
\]
Isoquants and Marginal Rate of Technical Substitution in Action

The Production Function

- $Q_1 = 100$ units of output
- $Q_2 = 200$ units of output
- $Q_3 = 300$ units of output

Capital Input

Labor Input

Increasing output

Substituting labor for capital
Diminishing Marginal Rate of Technical Substitution in Action

The Production Function

Capital Input (K)

ΔK = 3

ΔK = 1

Labor Input (L)

ΔL = -1

ΔL = -1

ΔL = -1

Slope (at C): \( \frac{ΔK}{ΔL} = -\frac{3}{1} = -3 = -MRTS_{KL} \)

Slope (at A): \( \frac{ΔK}{ΔL} = -\frac{1}{1} = -1 = -MRTS_{KL} \)

Q₀ = 100 units
Isocost and Changes in Isocost Lines

• Isocost
  - Combination of inputs that yield cost the same cost.
    \[ wL + rK = C \]
  or, re-arranging to the intercept-slope formulation:
    \[ K = \frac{C}{r} - \frac{w}{r}L \]

• Changes in isocosts
  - For given input prices, isocosts farther from the origin are associated with higher costs.
  - Changes in input prices change the slopes of isocost lines.
Isocost Line

The Production Function

\[ K = \frac{C}{r} - \frac{w}{r}L \]
Changes in the Isocost Line

The Production Function

Changes in the Isocost Line

Capital Input (K)

Less expensive input bundles

More expensive input bundles

Labor Input (L)
Changes in the Isocost Line

The Production Function

Due to increase in wage rate $w^1 > w^0$
Cost-Minimization Input Rule in Action

The Production Function

- Labor Input (L): $Q_I = 100$ units
- Capital Input (K): $MRTS_{KL} = \frac{w}{r}$

Diagram:
- Capital Input (K) vs. Labor Input (L)
- Points A, C1, and C2
- Slope: $\frac{C^2}{r}$, $\frac{C^1}{r}$, $\frac{C^2}{w}$, $\frac{C^1}{w}$

Equation:
- $\frac{C^1}{r} = \frac{C^2}{w}$
- $MRTS_{KL} = \frac{w}{r}$
Cost Minimization and the Cost-Minimizing Input Rule

• Cost minimization
  – Producing at the lowest possible cost.

• Cost-minimizing input rule
  – Produce at a given level of output where the marginal product per dollar spent is equal for all inputs:
    \[ \frac{MP_L}{w} = \frac{MP_K}{r} \]
  – Equivalently, a firm should employ inputs such that the marginal rate of technical substitution equals the ratio of input prices:
    \[ \frac{MP_L}{MP_K} = \frac{w}{r} \]
• Suppose that labor and capital are hired at a competitive wage of $10 and $25, respectively. If the marginal product of capital is 6 units and the marginal product of labor is 3 units, is the firm hiring the cost-minimizing units of capital and labor?

  – Since \( \frac{3}{10} > \frac{6}{25} \), the marginal product per dollar spent on labor exceeds the marginal product per dollar spent on capital.

  – The firm is not minimizing costs and should use fewer units of capital and more labor.
Optimal Input Substitution in Action

The Production Function

Capital Input (K)

Labor Input (L)

New cost-minimizing point due to higher wage

Initial point of cost minimization
The Cost Function

• Mathematical relationship that relates cost to the cost-minimizing output associated with an isoquant.

• Short-run costs
  – Fixed costs: $FC$
  – Sunk costs
  – Short-run variable costs: $VC(Q)$
  – Short-run total costs: $TC(Q) = FC + VC(Q)$

• Long-run costs
  – All costs are variable
  – No fixed costs
Short-Run Costs in Action

The Cost Function

\[ TC(Q) = FC + VC(Q) \]

Total costs
Variable costs
Fixed costs

Output

FC

0
Average and Marginal Costs

• Average costs
  – Average fixed: \( AFC = \frac{FC}{Q} \)
  – Average variable costs: \( AVC = \frac{VC(Q)}{Q} \)
  – Average total cost: \( ATC = \frac{C(Q)}{Q} \)

• Marginal cost
  – The (incremental) cost of producing an additional unit of output.
  – \( MC = \frac{\Delta C}{\Delta Q} \)
The Relationship between Average and Marginal Costs in Action

ATC, AVC, AFC and MC ($)

Minimum of ATC

Minimum of AVC

Output

ATC

AVC

AFC

MC

The Cost Function
Fixed and Sunk Costs

• Fixed costs
  – Cost that does not change with output.

• Sunk cost
  – Cost that is forever lost after it has been paid.

• Principle of Irrelevance of Sunk Costs
  – A decision maker should ignore sunk costs to maximize profits or minimize loses.
Long-Run Costs

• In the long run, all costs are variable since a manager is free to adjust levels of all inputs.

• Long-run average cost curve
  – A curve that defines the minimum average cost of producing alternative levels of output, allowing for optimal selection of both fixed and variable factors of production.
Long-Run Average Total Costs in Action

The Cost Function

LRAC ($) vs. Output

ATC₀, ATC₁, ATC₂, LRAC

Q*
Economies of Scale

• Economies of scale
  – Portion of the long-run average cost curve where long-run average costs decline as output increases.

• Diseconomies of scale
  – Portion of the long-run average cost curve where long-run average costs increase as output increases.

• Constant returns to scale
  – Portion of the long-run average cost curve that remains constant as output increases.
Economies and Diseconomies of Scale in Action

The Cost Function

LRAC ($)

- **Economies of scale**
- **Diseconomies of scale**

Output

0

$Q^*$
Constant Returns to Scale in Action

The Cost Function

LRAC ($) vs. Output

ATC<sub>1</sub>, ATC<sub>2</sub>, ATC<sub>3</sub>

LRAC
Multiple-Output Cost Function

• Economies of scope
  – Exist when the total cost of producing $Q_1$ and $Q_2$ together is less than the total cost of producing each of the type of output separately.
    \[ C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2) \]

• Cost complementarity
  – Exist when the marginal cost of producing one type of output decreases when the output of another good is increased.
    \[ \frac{\Delta MC_1(Q_1, Q_2)}{\Delta Q_2} < 0 \]
Multiple-Output Cost Function in Action

- Suppose a firm produces two goods and has cost function given by
  \[ C = 100 - 0.5Q_1Q_2 + (Q_1)^2 + (Q_2)^2 \]
- If the firm plans to produce 4 units of \( Q_1 \) and 6 units of \( Q_2 \)
  - Does this cost function exhibit cost complementarities?
    - Yes, cost complementarities exist since \( a = -0.5 < 0 \)
  - Does this cost function exhibit economies of scope?
    - Yes, economies of scope exist since \( 100 - 0.5(4)(6) > 0 \)
Conclusion

• To maximize profits (minimize costs) managers must use inputs such that the value of marginal product of each input reflects the price the firm must pay to employ the input.

• The optimal mix of inputs is achieved when $\frac{MRTS_{KL}}{w} = \frac{w}{r}$.

• Cost functions are the foundation for helping to determine profit-maximizing behavior in future chapters.
Cost-Minimization In Action

The Production Function

Labour Input (L) \[ Q_I = 100 \text{ units} \]

Capital Input (K)

\[ MRTS_{KL} = \frac{w}{r} \]

\( Q_I = 100 \text{ units} \)
Diminishing Marginal Rate of Technical Substitution in Action

The Production Function

Capital Input (K)

ΔK = −5

ΔK = −1

Labor Input (L)

ΔL = 1

ΔL = 1

A

B

C

D

Slope (at A): \( \frac{\Delta K}{\Delta L} = -\frac{5}{1} = -5 = -MRTS_{KL} \)

Slope (at C): \( \frac{\Delta K}{\Delta L} = -\frac{1}{1} = -1 = -MRTS_{KL} \)

\( Q_I = 100 \) units
Suppose that capital and labor are hired at a competitive wage of $10 and $20, respectively. If the marginal product of capital is 5 units and the marginal product of labor is 10 units, is the firm hiring the cost-minimizing units of capital and labor?

- Since \( \frac{5}{10} = \frac{10}{20} \Rightarrow \frac{MP_K}{r} = \frac{MP_L}{w} \), the cost-minimizing mix of capital and labor is being utilized.
Multiple-Output Cost Function in Action

• Suppose a firm produces two goods and has cost function given by
\[ C = 500 - 0.3Q_1Q_2 + (Q_1)^2 + (Q_2)^2 \]

• If the firm plans to produce 4 units of \( Q_1 \) and 6 units of \( Q_2 \)
  – Does this cost function exhibit cost complementarities?
    • Yes, cost complementarities exist since \( a = -0.3 < 0 \)
  – Does this cost function exhibit economies of scope?
    • Yes, economies of scope exist since
    \[ 500 - 0.3(4)(6) > 0 \]