12.4 Derivatives of Exponential Functions
12.5 Derivatives of Logarithmic Functions

**Derivative of \( e^x \) and \( e^{g(x)} \)**

\[
\frac{d}{dx}(e^x) = e^x
\]

\[
\frac{d}{dx}(e^{g(x)}) = e^{g(x)} \cdot g'(x)
\]

**Derivative of \( a^x \) and \( a^{g(x)} \)**

For any positive constant \( a \neq 1 \)

\[
\frac{d}{dx}(a^x) = (\ln a)a^x
\]

\[
\frac{d}{dx}(a^{g(x)}) = (\ln a)a^{g(x)} \cdot g'(x)
\]

**Derivative of \( \log_a x \), \( \log_a |x| \) and \( \log_a |g(x)| \)**

\[
\frac{d}{dx}(\log_a x) = \frac{1}{(\ln a)x}
\]

\[
\frac{d}{dx}(\log_a |x|) = \frac{1}{(\ln a)x}
\]

\[
\frac{d}{dx}(\log_a |g(x)|) = \frac{g'(x)}{(\ln a)g(x)}
\]

**Derivative of \( \ln(x) \), \( \ln| x | \) and \( \ln| g(x) | \)**

\[
\frac{d}{dx}(\ln(x)) = \frac{1}{x}
\]

\[
\frac{d}{dx}(\ln|x|) = \frac{1}{x}
\]

\[
\frac{d}{dx}(\ln|g(x)|) = \frac{g'(x)}{g(x)}
\]

**Logistic Function**

Sometimes a population or the sales of a product, will start growing slowly, then grow more rapidly, and then gradually level off and is best represented by a logistic function.

\[
G(t) = \frac{mG_0}{G_0 + (m - G_0)e^{-kt}}
\]

- \( t \) = time
- \( G_0 \) = initial number present
- \( m \) = maximum possible size of the population
- \( k \) = a positive constant
- \( G(t) \) = population at time \( t \)