

Transactive Energy Pricing in Power Distribution Systems

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Abstract—In the past years, demand response has been developed as a technique to tackle intermittency in the power generation of renewable generators. However, unpredictability of the responses of demand response providers to the price of electricity is a major drawback of relying on demand response programs. Transactive Energy is an emerging framework to address the above shortcoming. In this paper, a power market is developed for power distribution systems that enables power consumers to exchange transactive energy in real time operation of distribution systems. Furthermore, using law of supply and demand the real time value of transactive energy is derived.

Keywords: Transactive energy, demand response, renewable energy, energy market, distribution locational marginal price.

I. INTRODUCTION

Utilizing renewable resources of energy such as wind or solar energy is a promising approach to supply the growing demand for energy in the U.S. The energy produced by wind turbines and solar panels comes with a cost that is substantially less than the cost of energy production from fuel generators. Replacing the traditional power generators with renewable generators also results in reducing carbon emission and thereby helps to have a cleaner environment. However, one key challenge in utilization of renewable energy is the intermittency in the amount of energy from renewable generators, that unpredictably varies over the time. This challenge becomes bolder at higher levels of renewable energy penetration in the power grid, as is projected in the next following years [1].

In the past years, demand response has been developed as a technique to tackle the challenge discussed above. In demand response, power consumption of flexible electric loads are adjusted according to the availability of power generation, in contrast to the traditional setup in power grids where power generation is adjusted according to the demand for energy. Demand response is provided by power consumers with flexible loads such as air conditioners, heaters, electric pumps and electric vehicles. The adjustment of flexible loads' power consumptions can be carried out by two different mechanisms, i.e., direct load control and price responsiveness.

In direct load control, power consumers give control of their flexible loads to a central manager, who manages operation of the loads according to the preferences of the power consumers and power grid circumstances. One major drawback with direct load control mechanism is the privacy issues that it may cause for the consumers, as the private information of consumers will be disclosed to the central agent managing flexible loads. In price responsiveness mechanism, price of energy is sent to power consumers as a signal indicating circumstances of the power grid. The power consumers respond to the price signal so as to lower their electricity bills.

One major drawback with the price responsiveness mechanism is the unpredictability of the responses of demand response providers to the price signal, which causes the central agent to rely on statistical inferences about consumers' responses and plan for worst case scenario of consumers' responses [2].

Transactive energy is an emerging framework that addresses demand response shortcomings [2]. In contrast to demand response framework where reduction in the available energy from renewable generators triggers reaction of central agents, in transactive energy framework shrinkage of renewable energy triggers reaction of power consumers to the real time value of transactive energy. Consequently, real time value of transactive energy serves as an operational parameter to hold a dynamic balance between supply and demand of energy within the entire power grid [3].

The contribution of this paper is as follows:

- The real time value of transactive energy is calculated using the law of supply and demand.
- A distribution market is proposed in which power consumers can make real time energy transactions.
- The participation of consumers in the proposed distribution market is non-intrusive to their participation in the energy utility market.

The rest of this paper is organized as follows: System model is explained in Section II. Proposed distribution market is introduced in Section III. Numerical results are provided in Section IV. The paper is concluded in Section V.

II. SYSTEM MODEL

In this section, the operation of a power grid is discussed in terms of the energy markets and physical constraints of power distribution systems. Furthermore, it is explained how the distribution system is operated to deliver energy to consumers reliably. Finally, the current billing mechanism in energy utilities is discussed.

A. Energy Purchase from Energy Utilities

In regions with restructured electricity markets, including 32 States of the U.S., bulk amounts of energy are traded in wholesale electricity markets which are organized by regional Independent System Operators. Energy utilities as buyers of energy submit their energy demands to the wholesale markets, whereas bulk power generators specify their cost of productions as a function of the amount of energy that they may produce. The Independent System Operator clears the market and determines the lowest-cost power dispatch that minimizes total cost of production in supplying energy demand of energy utilities. As part of market clearing process, prices of energy at various nodes of the power grid are determined in a way

that social welfare is maximized and fairness to all market participants is guaranteed.

Each energy utility manages and operates a power distribution system, where bulk amounts of energy are received from the bulk transmission system and are distributed among power consumers. Before participating in wholesale markets, an energy utility estimates energy demand of the power consumers within its territory using prediction and forecasting techniques [4]. Using an electricity tariff, the energy utility translates the price of energy in wholesale markets to a price of electricity for power consumers. The price of energy set by electricity tariff varies over the time in a way that higher prices are set for hours of the day with higher energy demands. To calculate the consumers' electricity bills, the energy consumptions of consumers are metered by the electric meters that are installed at consumers' premises and are reported to the energy utility. For each consumer, the monthly electricity bill is calculated using the metered energy consumption of the consumer and the electricity tariff. In the setup discussed above, the power consumers can draw desirable amounts of energy from the power grid in real time operation of the grid, and pay for their energy consumptions at the end of billing cycles.

B. Power Flow Equations

The technical discussion in this paper is applicable to the whole or a smaller section of a power distribution system. Consider a section of power distribution system with N buses. The flows of real and reactive power in the power distribution lines are according to the following power flow equations:

$$P_k = \sum_{j=1}^N V_k V_j (G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j)) \forall k, \quad (1)$$

$$Q_k = \sum_{j=1}^N V_k V_j (G_{kj} \sin(\theta_k - \theta_j) - B_{kj} \cos(\theta_k - \theta_j)) \forall k, \quad (2)$$

where V_k , θ_k , P_k and Q_k , are voltage magnitude, phase angle, real power injection and reactive power injection at bus k , respectively. Also, G_{kj} and B_{kj} are negative of conductance and susceptance of the distribution line k-j. Each bus of the power distribution system can be of three types: The buses with electric loads are P-Q buses, the voltage regulated buses are P-V buses and a single reference bus is selected as the slack bus. Depending on the type of a bus, the voltage and power injection at the bus can be either known parameters or unknown variables. Namely, for a P-Q bus the real and reactive power injections to the bus are known parameters whereas voltage magnitude and phase angle at the bus are unknown variables. For a P-V bus, the voltage magnitude of the bus and real power injection to the bus are known parameters, whereas phase angle and reactive power injection to the bus are unknown variables. For the single bus that servers as the slack bus, voltage magnitude and phase angle are known parameters, whereas real and reactive power injections to the bus are unknown variables. Accordingly, we have:

$$V_k = V_{k,\text{nominal}} \quad \forall k \in \text{P-V and Slack Bus}, \quad (3)$$

$$\theta_k = \theta_{k,\text{nominal}} \quad \exists k = \text{Slack Bus}. \quad (4)$$

The equations (1) and (2) corresponding to P-Q buses, equations (1) and (3) corresponding to P-V buses, and equations (3) and (4) corresponding to the slack bus make up the power flow problem. The power flow problem is solved by Distribution System Operator to derive the state of power distribution system, including the voltages and power injections at various buses of the distribution system. The power flow problem is a non-linear system of equations and can be solved using Newton-Raphson method [5, Section 7.1].

C. Distribution System Operational Constraints

The voltages and power injections derived from the power flow problem should be checked to ensure that they fall within proper ranges. Namely, to operate the distribution system reliably the voltage magnitudes and phase angles at buses of the distribution system should fall within the following ranges:

$$V_{k,\text{Min}} \leq V_k \leq V_{k,\text{Max}} \quad \forall k, \quad (5)$$

$$\theta_{k,\text{Min}} \leq \theta_k \leq \theta_{k,\text{Max}} \quad \forall k. \quad (6)$$

Also, the power injection at P-V buses should fall within the following ranges:

$$P_{k,\text{min}} \leq P_k \leq P_{k,\text{max}} \quad \forall k \in \text{P-V buses}, \quad (7)$$

$$Q_{k,\text{min}} \leq Q_k \leq Q_{k,\text{max}} \quad \forall k \in \text{P-V buses}. \quad (8)$$

Finally, the real power flow in distribution lines should fall within the capacity of distribution lines:

$$|P_{kj}| \leq \text{Capacity of Distribution Line k-j}, \quad (9)$$

where

$$P_{kj} = V_k V_j (G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j)) - G_{k,j} V_k^2 \quad \forall k, j \in \text{Distribution Lines}. \quad (10)$$

If it turns out that one of the equations (6)-(9) is violated, relevant adjustments should be made to the topology or operational parameters of the distribution system using phase-shifting transformers or capacitor banks, to move the voltages, power injections and power flows to the proper ranges [6].

D. Power Distribution System State Estimation

Having access to the data on power consumption of consumers in real time operation of a distribution system, enables the Distribution System Operator to continuously update the parameters of power flow problem, solve the problem every few seconds, and adjust the power injections to the distribution system to supply energy demand of consumers in the most economical way. However, accessing the data on power consumptions of all consumers in real time operation of the distribution system may not be possible, because of lacking required Advanced Metering Infrastructure [7] or communication delays. In such circumstances, where real time data on power consumptions of consumers is not available, state estimation techniques [8] can be utilized to derive estimated values of voltages and power consumptions at various buses of the power distribution system.

In the state estimation process, voltage magnitudes and phase angles at various strategic locations of the distribution system are collected by electric meters or synchrophasors [9], and are reported to the Distribution System Operator. Using optimization techniques, a set of estimated values for voltage magnitudes and phase angles of the distribution system are selected in a way that, solution of power flow problem becomes close to the collected data. Whether directly collected from power consumers' premises, or indirectly estimated by the state estimation process, we assume that the voltage magnitudes and phase angles of the distribution system are known to Distribution System Operator.

III. DISTRIBUTION SYSTEM ENERGY MARKET

In this section, we introduce an energy market for power distribution systems, the operation of which is non-intrusive to the operation of energy utilities' markets. More precisely, the operation of a distribution system market neither impedes power consumers from purchasing monthly-billed energy from an energy utility, nor it enforces power consumers to purchase energy from the distribution system market. Rather, the power consumers can freely participate in the proposed distribution system market and make energy transactions that satisfy their energy demands. Before discussing details of energy transactions, we enumerate two characteristics of the distribution market.

First, the price of energy in distribution market reflects the value of energy arisen from law of supply and demand [10]. Second, the participants in the distribution market enjoy a price of electricity that varies according to the circumstances of the power grid and is known at real time operation of the power grid, in contrast to the price of energy set by energy utility that varies in time intervals with a length of minutes to hours. Such real time electricity pricing of energy in the distribution market, helps buyers to better manage their power consumption according to the instantaneous price of energy, and helps suppliers to adjust their energy offer according to the real time value of electricity.

A. Distribution Market Participation

Energy buyers in the distribution market are power consumers that purchase energy from the distribution market in addition to the energy that they may purchase from energy utility. The suppliers are demand response providers, energy storage units and renewable energy generators that provide energy to other participants. These different types of suppliers provide energy to the distribution market through different mechanisms. Namely, the demand response providers offer energy to the distribution market by lowering the energy consumption that they procure from the energy utility market. The energy storage units procure energy from the energy utility and inject it to the power distribution system following the energy transactions that they make in the distribution market. Finally, renewable generators offer their available energy to the distribution market before injecting power to the distribution system.

For simplifying the discussion, we assume that only a single power consumer may exist at each bus of the power distribution system. Each power consumer participating in the distribution market submits two price bids to the distribution market, one price bid corresponding to real power and the other price bid corresponding to reactive power. The price bid submitted by the market participant at bus k corresponding to real and reactive power are denoted by $\text{Bid}_{k,P}$ and $\text{Bid}_{k,Q}$, respectively. A positive price bid is for supplying energy and is submitted by a supplier. A negative price bid is for buying energy and is submitted by a buyer.

The real and reactive price bids submitted by suppliers specify the suppliers' cost of production of real and reactive power, respectively. On the other hand, real and reactive price bids submitted by buyers specify the highest prices at which buyers are still interested in purchasing energy from the distribution market. For instance, if the market participant at bus k submits a price bid of $\text{Bid}_{k,P} = 40$ \$/MW to the market, the market participant specifies that it asks for at least 40 \$/MW to inject real power to the distribution system. On the other hand, if the market participant submits a price bid of $\text{Bid}_{k,P} = -40$ \$/MW to the distribution market, it pays for energy at a price up to 40 \$/MW. The power consumers of the distribution system that don't wish to participate in the distribution market can either submit zero price bids or very large positive price bids.

B. Distribution Locational Marginal Prices

To preserve real time feature of the energy transactions at distribution market, the price bids submitted by market participants are updated every ΔT seconds, e.g. $\Delta T = 1$ second. At the beginning of a transaction cycle, market participants submit their updated price bids. The Distribution Market Operator collects the price bids from the participants, clears the market and sends out dispatch instructions to the suppliers and buyers with cleared bids. As part of this process, Distribution Market Operator determines the distribution locational marginal prices (D-LMPs) of energy at all buses of the distribution system, and uses the D-LMPs to calculate the payments to the suppliers and the charges to the buyers. The market clearing process is discussed in the next sections. It worth noting that, the choice of the length of transaction cycles ΔT depends on various factors including the rate of data transfer between Distribution System Operator and market participants. Lower ΔT can be chosen for power distribution systems with higher rates of data transfer between Distribution System Operator and market participants. The focus of the rest of this paper is on a single transaction cycle.

C. Differential Power Flow Equations

Let P_k and Q_k denote the real and reactive power injections to bus k at the beginning of a transaction cycle, respectively. Positive values of P_k and Q_k correspond to real and reactive power productions at bus k , respectively, whereas negative values of P_k and Q_k correspond to real and reactive power consumptions at bus k , respectively. The amount of transacted

energy at bus k of the distribution system is specified in terms of additional real power ΔP_k and reactive power ΔQ_k injections to bus k . Since the transaction cycle ΔT is small, real and reactive energy transactions are limited to maximum amounts of $\Delta P_{\max} \Delta T$ MW and $\Delta Q_{\max} \Delta T$ MVA per transaction, e.g. $\Delta P_{\max} = 0.001$ MW and $\Delta Q_{\max} = 0.001$ MVA:

$$\begin{aligned} 0 &\leq \text{sign}(\text{Bid}_{k,P}) \Delta P_k \leq \Delta P_{\max} \quad \forall k, \\ 0 &\leq \text{sign}(\text{Bid}_{k,Q}) \Delta Q_k \leq \Delta Q_{\max} \quad \forall k. \end{aligned} \quad (11)$$

In equation (11), $\text{sign}(\cdot)$ is the sign function, i.e., we have $\text{sign}(x) = 1$ for a positive x , $\text{sign}(x) = -1$ for a negative x , and $\text{sign}(x) = 0$ for $x = 0$. At the end of the transaction cycle, the real and reactive power injections to bus k will be $P_k + \Delta P_k$ and $Q_k + \Delta Q_k$, respectively.

Since the size of each energy transaction is small, the changes in voltage magnitudes ΔV_k and phase angles $\Delta \theta_k$ of the distribution system in a single transaction cycle is small as well. Accordingly, using the Taylor series [11, page 772] the power flow equations in (1) and (2) can be approximated by the following differential power flow equations:

$$\Delta P_k = \sum_{i=1}^N \frac{\partial P_k}{\partial V_i} \Delta V_i + \frac{\partial P_k}{\partial \theta_i} \Delta \theta_i, \quad (12)$$

$$\Delta Q_k = \sum_{i=1}^N \frac{\partial Q_k}{\partial V_i} \Delta V_i + \frac{\partial Q_k}{\partial \theta_i} \Delta \theta_i. \quad (13)$$

As it was specified in Section II-D, we assume that at the beginning of the transaction cycle, the current state of the power distribution system including voltage magnitudes V_k and phase angles θ_k are known to the Distribution System Operator. Therefore, the partial derivatives in the differential power flow equations (12) and (13) can be readily obtained from the following equations:

$$\begin{aligned} \frac{\partial P_k}{\partial V_i} &= \begin{cases} V_k (G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)) & i \neq k \\ P_k / V_k + V_k G_{kk} & i = k \end{cases} \\ \frac{\partial P_k}{\partial \theta_i} &= \begin{cases} V_k V_i (G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)) & i \neq k \\ -Q_k - V_k^2 B_{kk} & i = k \end{cases} \\ \frac{\partial Q_k}{\partial V_i} &= \begin{cases} V_k (G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)) & i \neq k \\ Q_k / V_k - V_k B_{kk} & i = k \end{cases} \\ \frac{\partial Q_k}{\partial \theta_i} &= \begin{cases} -V_k V_i (G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)) & i \neq k \\ P_k - V_k^2 G_{kk} & i = k \end{cases} \end{aligned} \quad (14)$$

D. Real Time Operational Constraints in Distribution Systems

Applying the Taylor series to equation (10), the differential real power flow in distribution line k - j is obtained as follows:

$$\begin{aligned} \Delta P_{kj} &= -2V_k G_{kj} \Delta V_k - \frac{1}{V_k} \frac{\partial Q_k}{\partial \theta_j} \Delta V_k + \frac{1}{V_j} \frac{\partial Q_k}{\partial \theta_j} \Delta V_j \\ &\quad - V_j \frac{\partial Q_k}{\partial V_j} \Delta \theta_k + V_j \frac{\partial Q_k}{\partial V_j} \Delta \theta_j. \end{aligned} \quad (15)$$

In formulating ΔP_{kj} in (15), partial derivatives in (14) are used just for shortening the expression of ΔP_{kj} and saving

space. In making energy transactions in the distribution market, following constraint should be preserved to ensure reliable operation of distribution lines:

$$|P_{kj} + \Delta P_{kj}| \leq \text{Capacity of Line } k\text{-}j \quad \forall k, j \in \text{Lines} \quad (16)$$

In (16), P_{kj} is a parameter that can be calculated using (9). Also, the right hand side of (16) is a given parameter known to Distribution System Operator. Furthermore, to preserve the voltage magnitudes and phase angles within the safe ranges given by (5) and (6), the following equations should be taken into account in making energy transactions in the distribution market:

$$\begin{aligned} V_{\text{Min}} &\leq V_k + \Delta V_k \leq V_{\text{Max}} \quad \forall k, \\ \theta_{\text{Min}} &\leq \theta_k + \Delta \theta_k \leq \theta_{\text{Max}} \quad \forall k. \end{aligned} \quad (17)$$

In (17), V_k and θ_k are parameters known to Distribution System Operator; see Section II-D.

E. Social Welfare Maximization

For clearing the distribution market, Distribution System Operator solves the following optimization problem:

$$\begin{aligned} &\text{Maximize}_{\Delta P_k, \Delta Q_k, \Delta V_k, \Delta \theta_k} \Delta T \sum_{k=1}^N -|\text{Bid}_{k,P}| \Delta P_k - |\text{Bid}_{k,Q}| \Delta Q_k \\ &\text{Subject to} \quad (12) - (13) \text{ and } (15) - (17) \end{aligned} \quad (18)$$

The objective function in problem (18) is the social welfare for the market participants, and accordingly problem (18) maximizes the social welfare. The problem (18) is a linear optimization problem that can be solved by CVX. Also, the optimization variables in problem (18) are ΔP_k , ΔQ_k , ΔV_k and $\Delta \theta_k$. The optimal solutions ΔP_k^* and ΔQ_k^* are transmitted to market participants as dispatch instructions to increase or decrease the power consumption at the end of the transaction cycle. The optimal solutions ΔV_k^* and $\Delta \theta_k^*$ are checked against the estimated or metered voltages magnitudes and phase angles to ensure that state of the distribution system is changed according to the dispatch instructions.

F. Real Time Value of Transactive Energy

As part of the market clearing process, prices of energy at all buses of the distribution system should be determined. In wholesale energy markets, the price of energy at a node of bulk transmission system is set as the marginal cost of production in supplying the next additional Mega-Watt of energy demand at that node. Analogous to energy pricing in bulk transmission system, the price of real and reactive energy at bus k of the distribution system are set as the marginal cost of production in supplying the next Mega-Watt and Mega-Volt-Ampere power demands at bus k , respectively. Mathematically, Distribution Locational Marginal Price of Real Power D-LMP $_{k,P}$ at bus k is defined as follows:

$$\begin{aligned} \text{D-LMP}_{k,P} &\triangleq \\ &\frac{\sum_{k=1}^N \text{Bid}_{k,P} (\Delta \tilde{P}_k - \Delta P_k^*) + \text{Bid}_{k,Q} (\Delta \tilde{Q}_k - \Delta Q_k^*)}{\epsilon} \quad \forall k, \end{aligned} \quad (19)$$

where $\Delta\hat{P}_k$ and $\Delta\hat{Q}_k$ are solution of the problem (18) for the case where an additional amount of $\epsilon \ll 1$ real power is drawn from bus k . Similarly, Distribution Locational Marginal Price of Reactive power $D-LMP_{k,Q}$ at bus k is defined as follows:

$$D-LMP_{k,Q} \triangleq \frac{\sum_{k=1}^N \text{Bid}_{k,P}(\Delta\hat{P}_k - \Delta P_k^*) + \text{Bid}_{k,Q}(\Delta\hat{Q}_k - \Delta Q_k^*)}{\epsilon} \quad \forall k, \quad (20)$$

where $\Delta\hat{P}_k$ and $\Delta\hat{Q}_k$ are solution of the problem (18) for the case where an additional amount of $\epsilon \ll 1$ reactive power is drawn from bus k .

The following theorem provides a method to readily obtain Distribution Locational Marginal Prices from solution of problem (18):

Theorem 1: For each bus k of the power distribution system, $D-LMP_{k,P}$ and $D-LMP_{k,Q}$ defined by equations (19) and (20) are the dual variables in problem (18) corresponding to constraints (12) and (13), respectively.

Theorem 1 can be proved by reformulating problem (18) to the standard form of a linear optimization problem, and using the relation between dual variables and sensitivity of the optimal objective function of the problem to variations in right-hand-side of the constraints [12, Page 17]. Proof of Theorem 1 is skipped to preserve page limit of the paper. Considering that linear optimization solvers calculate dual variables as part of their solution algorithms, Theorem 1 indicates that $D-LMP_{k,P}$ and $D-LMP_{k,Q}$ are readily available after solving problem (18). Finally, we note that the payment to the market participant at bus k is as follows:

$$\text{Payment} = \Delta T(D-LMP_{k,P}\Delta P_k^* + D-LMP_{k,Q}\Delta Q_k^*) \quad \forall k, \quad (21)$$

where a negative payment for participant k is a charge to participant k .

IV. NUMERICAL RESULTS

In this Section, numerical results are provided to show the effectiveness of the proposed distribution market in carrying out energy exchanges between power consumers. Simulations are conducted on IEEE 33-bus system as the power distribution system. The length of transaction cycles is chosen as $\Delta T = 1$ seconds. The maximum power transactions allowed in a single transaction cycle are $\Delta P_{\max} = 0.001$ MW and $\Delta Q_{\max} = 0.001$ MVA. The simulations are performed using a computer with a CPU @2.60 GHz and 16 GB RAM.

The power consumers at buses 1, 2, 5, 6, 10 and 12 in IEEE 33-bus system participate in distribution market as suppliers with reactive power price bids of zeros, and initial real power price bids of 10, 5, 6, 7 and 9 \$/MW, respectively. The suppliers' real power price bids in the subsequent transaction cycles vary as a linear function of their transacted energy in distribution market with a slope of 2 \$/MW. The power consumer at bus 17 participates as a buyer with a reactive price bid of zero and an initial real power price of -6 \$/MW. In the subsequent transaction cycles, the buyer's real power price bid varies as a linear function of the buyer's transacted

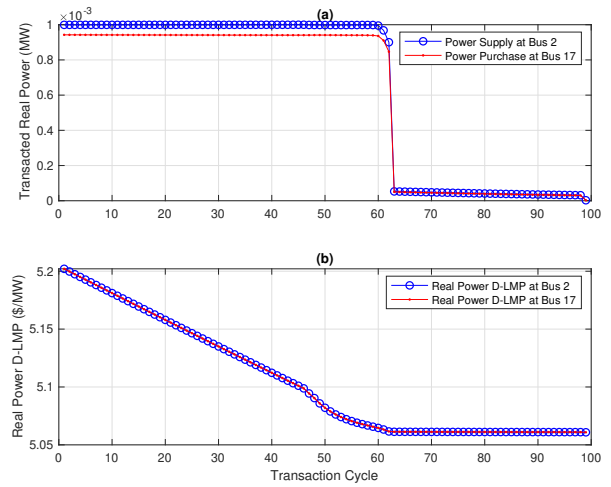


Fig. 1: Energy transactions between market participants at Bus 2 and Bus 17 of IEEE 33-Bus System. a) real power transactions b) real power D-LMPs.

energy in distribution market with a slope of 2 \$/MW. Fig. 1 shows the transacted energies between the supplier at bus 2 and the buyer at bus 17. From Fig. 1, the market participants can exchange energy at distribution market without intruding the operation of power distribution system.

V. CONCLUSION

In this paper, a distribution market was proposed that determines the real time value of transactive energy, using the law of supply and demand. The proposed distribution market enables the power consumers of distribution system to make energy transactions in real time, without intervening energy purchase from energy utilities.

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