Strategic Bidding in Electricity Markets: A Power Grid Partitioning Approach

Lixia Chen[1], Donghan Feng[2]
Department of Electrical Engineering
Shanghai Jiao Tong University
Shanghai, China
Email: [1]vogue@sjtu.edu.cn, [2]seed@sjtu.edu.cn

Mahdi Ghamkhari
Department of Electrical and Computer Enigneering
University of Louisiana at Lafayette
Lafayette, LA, USA
mahdi.ghamkhari@louisiana.edu

***Abstract*—By bidding strategically, a producer firm owning several generators aims to maximize its profit by submitting optimum bids to the wholesale electricity markets. For a bulk power producer that influences the price of electricity, i.e. a price-maker producer, strategic bidding problem appears as a highly non-convex and challenging optimization problem. The computation time of the state-of-the-art solution method for strategic bidding problem increases linearly as the number of random scenarios or scheduling horizon increases. However the computation time of the state-of-the-art approach increases significantly as the size of power grid increases. This paper proposes a partitioning method that when is combined with the state-of-the-art solution method results in considerable reduction in computation time.**

Keywords: Strategic bidding, electricity market, equilibrium constraints, convex optimization, computation time.

NOMENCLATURE

$\mathbb{R}$	Set of real numbers
$\mathbb{S}$	Set of symmetric matrices
a	Vector of price bids for strategic generators
P_G	Vector of power generations
P_D	Vector of power demands
θ	Vector of phase angels of power grid
λ	Vector of locational marginal prices
σ, δ, ζ,	Vectors of dual variables corresponding
ξ, ϕ, ψ	to inequalities in economic dispatch problem
x	Column vector of all variables in (1)
i, j	Indices for linear inequalities in (1)
z	Index for quadratic equalities in (1)
n	Length of the vector x
F	Symmetric matrice of parameters in $\mathbb{S}^n$
f, p, q, q'	Vectors of parameters in $\mathbb{R}^n$
R^{Aux}	Vector of linearly independent variables
l, r	Indices for the left and right sub-grids
$(\cdot)^T$	Transpose of a vector or a matrix
$*$	Point-wise multiplication of vectors
$\times$	Matrix multiplication
$\mathbf{I}$	Identity Matrix
$\mathbf{0}$	A column vector or a matrix with zero entries
$trace(\cdot)$	Trace of a matrix
$\succeq$	Matrix inequality

I. INTRODUCTION

Bulk power generators bid in the wholesale electricity markets for selling their energy production. Each submitted bid is comprised of two components, namely an energy bid and a price bid. The energy bid is the maximum amount of power that a generator may produce, and the price bid is a suggested price for the produced energy. The Independent System Operator (ISO) collects the submitted bids and clears the market by indicating the amount of energy that generators should produce and the associated price for the produced energy. In accomplishing this task, the ISO solves a linear optimization problem known as *Economic Dispatch problem*.

For a generator, profit is revenue minus cost of generation, where the revenue is energy production of the generator times the price of energy. In bidding strategically, a producer firm owning several generators aims to maximize total profit of its generator units by submitting optimal bids to the electricity market. Since the prices of energy come from Economic Dispatch problem, strategic bidding problem boils down to a bi-level optimization problem where the lower level problem is the Economic Dispatch problem and the higher level problem is a profit maximization problem.

The common practice to solve such a bi-level optimization problem is to incorporate the KKT optimality conditions of the lower level problem as a set of constraints in upper level problem. The resulted optimization problem is in the form of a Mathematical Problem with Equilibrium Constraints (MPEC) in which the non-convexity is due to the quadratic slackness constraints. The MPEC reformulation of strategic bidding problem is a very hard optimization problem to solve. The solution approaches that are proposed by [1] and [2] result in a very poor profit value for the strategic firm. The prior work [3] proposes a solution approach based on Mixed Integer Linear programming (MILP) that is analytically guaranteed to converge and give the optimal solution of strategic bidding problem. However, in practical cases the approach in [3] doesn't converge due to its drastic computation time. The approach in [4] gives a close-to-optimal profit value for the strategic firm, and comes with a computation time that is much less than that of the approach in [3]. More precisely, the approach in [4] comes with a computation time that grows *linearly* as the number of scenarios or time slots in strategic bidding problem increases. However, computation time of the approach in [4] increases significantly as the size of power grid increases. Specially, [4] explains:

"While the [solution method in] [4] takes a major leap in solving strategic bidding problems in nodal electricity markets compared to the [...] MILP-based approaches, it still faces some limitations that could be addressed in future follow up studies. For example, it appears that the proposed method is well-capable of handling the increases in the number of time slots and the number of random scenarios. However, it is still not fully capable of handling the increases in the number of buses."

This paper addresses the above issue by utilizing a partitioning technique along with the approach in [4]. As a result, computation time of the proposed approach is considerably less than that of the approach in [4], as is shown numerically by simulations on IEEE 57-bus system.

The rest of this paper is organized as follows: Strategic bidding problem is discussed in Section II. The solution approach in [4] is explained in Section III. The proposed solution approach in this paper is presented in Section IV. The performance of the proposed solution approach is discussed in Section V. The paper is concluded in Section VI.

II. Strategic Bidding Problem

The optimization problem that the firm of strategic generators should solve is in the following general form [4]:

$$\begin{aligned} \underset{x}{\text{maximize}} \quad & x^T F x + 2 f^T x \\ \text{subject to} \quad & p_i^T x + p_{i0} \geq 0 \qquad \forall i \\ & (q_z^T x + q_{z0})({q'_z}^T x + q'_{z0}) = 0 \quad \forall z. \end{aligned} \tag{1}$$

For each z, the second constraint in problem (1) is a quadratic slackness constraint that indicates the production of linear terms $q_z^T x + q_{z0}$ and ${q'_z}^T x + q'_{z0}$ should be zero. The problem in (1) is a non-convex optimization problem, because the quadratic objective function [5, Example 4.5] and the slackness constraints are all non-convex functions of optimization variable x. In this section, we explain the process of reformulating strategic bidding problem to the general form of (1) on a 3-bus power grid.

A. Economic Dispatch Problem

Consider the 3-bus power grid in Fig. 1 which includes one load, two generators and one strategic generator. The reactances and power capacity of transmission lines are assumed to be 1 and 0.2 per unit, respectively. The constraints in Economic Dispatch problem are as follows:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} P_G - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} P_D - \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \theta = \mathbf{0} \quad : \lambda, \tag{2}$$

$$\begin{bmatrix} 0 & 0 \end{bmatrix}^T \leq P_G \leq \begin{bmatrix} 0.5 & 0.3 \end{bmatrix}^T \quad : \sigma,\ \delta, \tag{3}$$

$$0 \leq P_D \leq 0.5 \quad : \zeta,\ \xi, \tag{4}$$

$$-\begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} \leq \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \theta \leq \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} \quad : \phi,\ \psi. \tag{5}$$

Equation (2) enforces power balance between generation and consumption at three nodes of the power grid. Also, equations

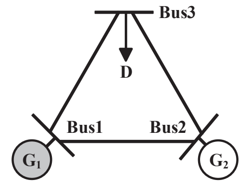

Fig. 1: A 3-bus power grid.

(3)-(5) enforces upper and lower bounds on power generation of the generators, power consumption of the load and power flow in the transmission lines, respectively. For each constraint, the optimization vector after the colon denotes the vector of dual variables corresponding to that constraint.

Generators and loads submit a price bid to the electricity market, as a suggested price for the electricity that they produce or consume. The price bids for the load and non-strategic generator in Fig. 1 are parameters with numerical values of 50 and 20, respectively. Let, a denote the price bid for the strategic generator in Fig 1. The ISO collects all the bids from generators and loads, and clears the market by solving the following Economic Dispatch problem [4]:

$$\begin{aligned} \min \quad & \begin{bmatrix} a \\ 20 \end{bmatrix}^T P_G - 50\, P_D \\ \text{subject to} \quad & (2) - (5) \end{aligned} \tag{6}$$

The solution of problem (6) determines the amount of power that each generator should produce. The price of electricity at each node of the power grid is the dual variable corresponding to the power balance constraint (2) at that node [3].

B. Strategic Bidding Problem

The strategic generator at node 3 seeks to maximize its profit by solving the following strategic bidding problem [4]:

$$\begin{aligned} \underset{\substack{a, P_G, P_D, \phi \\ \theta, \lambda, \sigma, \delta, \zeta, \xi, \psi}}{\text{maximize}} \quad & \lambda^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} P_G - 30 \begin{bmatrix} 1 & 0 \end{bmatrix} P_G \\ \text{subject to} \quad & \begin{pmatrix} P_G, P_D \\ \phi, \theta, \lambda, \sigma \\ \delta, \zeta, \xi, \psi \end{pmatrix} = \quad \text{argmin} \quad \begin{bmatrix} a \\ 20 \end{bmatrix}^T P_G - 50\, P_D \\ & \qquad\qquad\qquad \text{subject to} \quad (2) - (5). \end{aligned} \tag{7}$$

The first and second terms in the objective function of problem (7) are the revenue and cost of generation for the strategic generator, respectively. Since problem (6) is incorporated as a constraint in problem (7), problem (7) is a *bi-level* optimization problem.

The first step to solve problem (7) is to transform it to a single level optimization problem [3]. This is accomplished by replacing the lower level problem in (7) with its equivalent KKT conditions. The KKT conditions of problem (6) are comprised of constraints (2)-(5) and the following constraints:

$$\begin{bmatrix} a \\ 20 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \lambda - \sigma + \delta = \mathbf{0}, \tag{8}$$

$$50 - \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \lambda - \zeta + \xi = \mathbf{0}, \tag{9}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \lambda + \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} (\phi - \psi) = \mathbf{0}, \tag{10}$$

$$\sigma * P_G = \mathbf{0}, \tag{11}$$

$$\delta * (\begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix} - P_G) = \mathbf{0}, \tag{12}$$

$$\zeta * (P_D) = 0, \tag{13}$$

$$\xi * (0.5 - P_D) = 0, \tag{14}$$

$$\phi * \left(\begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \theta \right) = \mathbf{0}, \tag{15}$$

$$\psi * \left(\begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \theta \right) = \mathbf{0}, \tag{16}$$

$$\sigma \geq \mathbf{0}, \ \zeta \geq \mathbf{0}, \ \phi \geq \mathbf{0}, \tag{17}$$

$$\delta \geq \mathbf{0}, \ \xi \geq \mathbf{0}, \ \psi \geq \mathbf{0}. \tag{18}$$

Replacing the lower level problem, i.e. problem (6), in the bi-level optimization problem (7) with its equivalent KKT conditions, i.e. (2)-(5) and (8)-(18), results in the following optimization problem (7):

$$\begin{aligned} \underset{\substack{a, P_G, P_D, \phi \\ \theta, \lambda, \sigma, \delta, \zeta, \xi, \psi}}{\text{maximize}} \quad & \lambda^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} P_G - 30 \begin{bmatrix} 1 & 0 \end{bmatrix} P_G \\ \text{subject to} \quad & (2) - (5) \text{ and } (8) - (18), \end{aligned} \tag{19}$$

the solution of which is equal to the solution of problem (7). The problem (19) is in the form of a *Mathematical Problem with Equilibrium Constraints* (MPEC).

C. Eliminating Linear Equality Constraints

The set of Linear equality constraints in an optimization problem form a system of linear equations that can be solved and eliminated from the optimization problem in systematic way [5, Section 4.1.3] and [6, pp. 46]. Eliminating these constraints from the optimization problem doesn't change the solution of the problem but may be effective in reducing the computation time of solving the problem, as is the case for problem (19) [4]. In this section, we explain the process of eliminating linear equality constraints (2) and (8)-(10) from problem (19). The constraint (2) can be rewritten as follows:

$$\begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 1 \\ 0 & 1 & 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} P_G \\ P_D \\ \theta \end{bmatrix} = \mathbf{0} \tag{20}$$

For every P_G, P_D and θ, constraint (20) is a homogenous system of linear equations [7, Section 3.6]. Conseuently, constraint (20) holds if and only if the vector $\begin{bmatrix} P_G & P_D & \theta \end{bmatrix}$ falls within the null space [8] of the first matrix in the left hand side of (20). Therefore, constraint (20) can be equivalently rewritten as:

$$\begin{bmatrix} P_G \\ P_D \\ \theta \end{bmatrix} = \begin{bmatrix} 0.63 & -0.37 & -0.26 \\ -0.26 & 0.63 & -0.37 \\ 0.37 & 0.26 & -0.63 \\ 0.54 & 0.21 & 0.25 \\ 0.25 & 0.54 & 0.21 \\ 0.21 & 0.25 & 0.54 \end{bmatrix} R^{Aux}_{3\times1}, \tag{21}$$

where $R^{Aux}_{3\times1} \in \mathbb{R}^{3\times1}$ is a vector of linearly independent auxiliary variables. Equation (21) can be rewritten as the combination of the following three equations which give explicit expressions for P_G, P_D and θ as linear functions of $R^{Aux}_{3\times1}$:

$$P_G = \begin{bmatrix} 0.63 & -0.37 & -0.26 \\ -0.26 & 0.63 & -0.37 \end{bmatrix} R^{Aux}_{3\times1}, \tag{22}$$

$$P_D = \begin{bmatrix} 0.37 & 0.26 & -0.63 \end{bmatrix} R^{Aux}_{3\times1}, \tag{23}$$

$$\theta = \begin{bmatrix} 0.54 & 0.21 & 0.25 \\ 0.25 & 0.54 & 0.21 \\ 0.21 & 0.25 & 0.54 \end{bmatrix} R^{Aux}_{3\times1}. \tag{24}$$

Similarly, equation (10) can be equivalently rewritten as:

$$\begin{bmatrix} 2 & -1 & -1 & 1 & 1 & 0 & -1 & -1 & 0 \\ -1 & 2 & -1 & -1 & 0 & 1 & 1 & 0 & -1 \\ -1 & -1 & 2 & 0 & -1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda \\ \phi \\ \psi \end{bmatrix} = \mathbf{0}. \tag{25}$$

From (25), variables λ, ϕ and ψ can be rewritten as follows:

$$\lambda = \begin{bmatrix} 0.72 & -0.15 & -0.14 & 0.06 & 0.15 & 0.14 & -0.06 \\ 0.48 & 0.35 & 0.07 & -0.24 & -0.35 & -0.07 & 0.24 \\ 0.42 & 0.12 & 0.35 & 0.27 & -0.12 & -0.35 & -0.27 \end{bmatrix} R^{Aux}_{7\times1}, \tag{26}$$

$$\phi = \begin{bmatrix} -0.14 & 0.91 & -0.07 & 0.01 & 0.09 & 0.07 & -0.01 \\ -0.13 & -0.03 & 0.91 & -0.06 & 0.03 & 0.09 & 0.06 \\ -0.06 & 0.05 & -0.03 & 0.92 & -0.05 & 0.03 & 0.08 \end{bmatrix} R^{Aux}_{7\times1}, \tag{27}$$

$$\psi = \begin{bmatrix} 0.14 & 0.09 & 0.07 & -0.01 & 0.91 & -0.07 & 0.01 \\ 0.13 & 0.03 & 0.09 & 0.06 & -0.03 & 0.91 & -0.06 \\ 0.06 & -0.05 & 0.03 & 0.08 & 0.05 & -0.03 & 0.92 \end{bmatrix} R^{Aux}_{7\times1}, \tag{28}$$

where $R^{Aux}_{7\times1} \in \mathbb{R}^{7\times1}$ is a vector of linearly independent auxiliary variables. Finally, equations (8) and (9) can be eliminated by a rearrangement of their terms and substituting right hand side of (26) for λ:

$$\begin{aligned} \delta = & -\begin{bmatrix} a \\ 20 \end{bmatrix} + \sigma + \\ & \begin{bmatrix} 0.72 & -0.15 & -0.14 & 0.06 & 0.15 & 0.14 & -0.06 \\ 0.48 & 0.35 & 0.07 & -0.24 & -0.35 & -0.07 & 0.24 \end{bmatrix} R^{Aux}_{7\times1}, \end{aligned} \tag{29}$$

$$\begin{aligned} \xi = & -50 + \zeta + \\ & \begin{bmatrix} 0.42 & 0.12 & 0.35 & 0.27 & -0.12 & -0.35 & -0.27 \end{bmatrix} R^{Aux}_{7\times1}. \end{aligned} \tag{30}$$

Eliminating the linear equality constraints from problem (19) is comprised of two steps. First, the constraints (2) and (8)-(10) are removed from the problem (19). Second, the optimization variables P_G, P_D, θ, λ, ϕ, ψ in problem (19) are

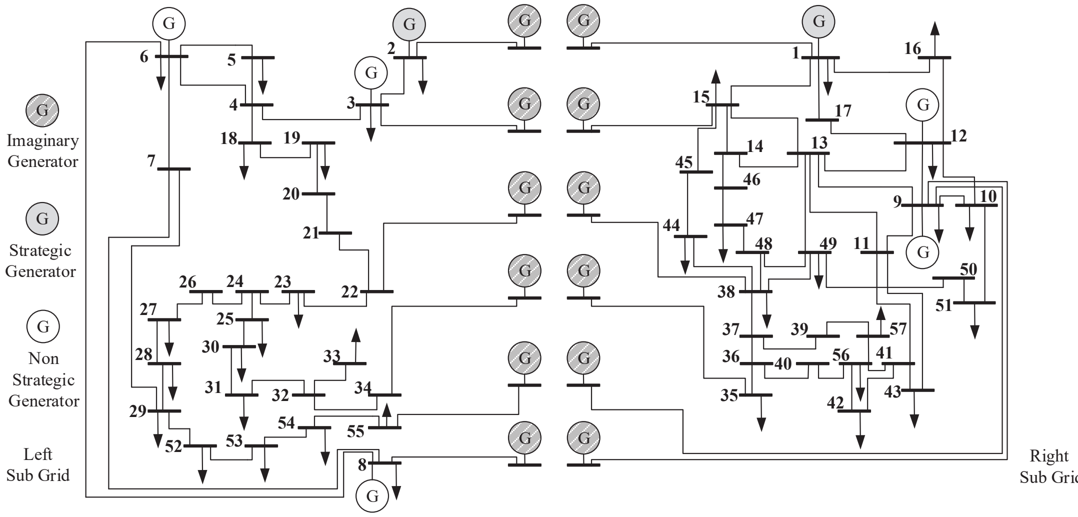

Fig. 2: IEEE standard 57-bus system is divided into two sub-grids. The transmission lines between two sub-grids are replaced with imaginary generators.

replaced with their equivalent expressions given by (21), (29) and (30). For instance, replacing P_G and σ in (12) with their equivalent expressions in (22) and (29) gives the following constraint that is equivalent to constraint (12):

$$\left(-\begin{bmatrix}a\\20\end{bmatrix}+\sigma+\begin{bmatrix}0.72 & -0.15 & -0.14 & 0.06 & 0.15 & 0.14 & -0.06\\0.48 & 0.35 & 0.07 & -0.24 & -0.35 & -0.07 & 0.24\end{bmatrix}R^{Aux}_{7\times1}\right)*\left(\begin{bmatrix}0.5\\0.3\end{bmatrix}-\begin{bmatrix}-0.63 & 0.37 & 0.26\\0.26 & -0.63 & 0.37\end{bmatrix}R^{Aux}_{3\times1}\right)=\begin{bmatrix}0\\0\end{bmatrix}. \quad (31)$$

The optimization problem that is resulted from the above two steps is equivalent to problem (19), but includes only a, σ, ζ, $R^{Aux}_{3\times1}$ and $R^{Aux}_{7\times1}$ as the optimization variables.

D. Strategic Bidding Problem in the Standard Form

After eliminating the linear equality constraints from strategic bidding problem, the optimization variables in the resulted problem can be stacked in a single optimization vector:

$$x \triangleq \left[a^T\ \sigma^T\ \zeta^T\ {R^{3\times1}_{Aux}}^T\ {R^{7\times1}_{Aux}}^T\right]^T. \quad (32)$$

The final step in converting the resulted optimization problem to the standard form of (1) is to rewrite the objective function and constraints of the resulted problem in terms of optimization vector x. For illustration of this process, consider the equation (31) that is comprised of two separate constraints. The first constraint in (31) can be rewritten in the form of last constraint in problem (1) by choosing $q_0 = 0$, $q'_0 = 0.5$ and:

$$q^T = -\left[\mathbf{I}_{1\times1}\ \mathbf{0}_{1\times13}\right]+\left[\mathbf{0}_{1\times1}\ \mathbf{I}_{1\times1}\ \mathbf{0}_{1\times12}\right]+\left[0.72\ -0.15\ -0.14\ 0.06\ 0.15\ 0.14\ -0.06\right]\left[\mathbf{0}_{7\times7}\ I_{7\times7}\right], \quad (33)$$

$$q'^T = -\left[-0.63\ 0.37\ 0.26\right]\left[\mathbf{0}_{3\times4}\ \mathbf{I}_{3\times3}\ \mathbf{0}_{3\times7}\right]. \quad (34)$$

III. Strategic Bidding in a Convex Optimization Framework

This Section summarizes the solution method proposed in [4] for the strategic bidding problem.

A. A Convex Optimization Framework

The optimal objective function of the following *convex* optimization problem is very close to the optimal objective function of problem (1) [4]:

$$\begin{aligned}
&\underset{X\in\mathbb{S}^{n+1}}{\textbf{maximize}} \quad trace\left(\begin{bmatrix}0 & f^T\\ f & F\end{bmatrix}X\right)\\
&\textbf{subject to}\\
&X_{11}=1\\
&trace\left(\begin{bmatrix}p_{i0} & p_i^T/2\\ p_i/2 & \mathbf{0}\end{bmatrix}X\right)\geq 0 \qquad \forall i\\
&trace\left(\begin{bmatrix}p_{i0}\\ p_i\end{bmatrix}\begin{bmatrix}p_{j0}\\ p_j\end{bmatrix}^T X\right)\geq 0 \qquad \forall i,j\\
&trace\left(\begin{bmatrix}q_{z0}q'_{z0} & q_{z0}{q'_z}^T\\ q'_{z0}q_z & q_z{q'_z}^T\end{bmatrix}X\right)=0 \quad \forall z\\
&X\succeq 0.
\end{aligned}$$

In problem (35), the optimization variable X is a positive semi-definite matrix. The problem (35) is a semi-definite program (SDP) [9] and can be solved using Mosek [10]. Let $X^\star$ denote the optimal solution of problem (35). The first column of $X^\star$ provides a good approximation for the solution of problem (1) [4]:

$$x^\star = \left[\mathbf{0}_{n\times1}\ I_{n\times n}\right]X^\star\left[1\ \mathbf{0}_{n\times1}\right]^T. \quad (35)$$

More precisely, numerical value of objective function of optimization problem (1) for $x = x^\star$ is very close to the optimal solution of problem (1). Still, $x^\star$ doesn't satisfy all the constraints in problem (1), i.e. $x^\star$ is not a feasible solution of problem (1). In the next section, it is explained how this issue can be resolved by systematically projecting $x^\star$ from (35) into the feasible space of constraints in problem (1).

B. Projecting Approximated Solution to the Feasible Set

The last constraint in problem (1) indicates that, for each z the solution of strategic bidding problem x should fall within one of the two affine spaces [5, Section 2.1] defined by the following two linear equations:

$$q_z^T x + q_{z0} = 0, \tag{36}$$

or

$$q_z'^T x + q'_{z0} = 0. \tag{37}$$

The solution given by (35) is used to determine which one of the spaces defined by (36) and (37) contains the solution of problem (1). If the solution $x^\star$ from (35) satisfies the following condition [4]:

$$|q_z^T x^\star + q_{z0}| \leq \epsilon = 0.1 \quad \text{and} \quad |q_z'^T x^\star + q'_{z0}| \geq \Delta = 1, \tag{38}$$

the affine space defined by (36) is likely to contain the solution of problem (1). Therefore, we can replace the non-convex constraint in (1) corresponding to z with the linear constraint given by (36). Similarly, if the solution $x^\star$ from (35) satisfies the following condition:

$$|q_z^T x^\star + q_{z0}| \geq \Delta = 1 \quad \text{and} \quad |q_z'^T x^\star + q'_{z0}| \leq \epsilon = 0.1, \tag{39}$$

the affine space defined by (37) is likely to contain the solution of problem (1). Therefore, we can replace the non-convex constraint in (1) corresponding to z with the linear constraint given by (37).

The problem that is obtained after the above process includes fewer number of non-convex quadratic constraints compared to the original problem in (1), and can be solved quickly for a guaranteed global solution using the MILP approach in [3]. In some cases, the problem that is obtained from the above process may not be feasible, but this issue can be easily resolved by setting ϵ to a smaller value, e.g., $\epsilon = 0.09$, $\epsilon = 0.08$, ... and repeating the above process until a feasible solution is derived. An algorithmic representation of the above process is presented in Algorithm 1 in [4]. Nevertheless, the solution from the above process provides a feasible solution for problem (1) that results in a close-to-optimal profit value for the strategic firm.

C. Strategic Bidding over a Time Horizon under Uncertainty

In practice, the strategic firm doesn't know the other generators' price bids before the market is cleared. The strategic firm only knows a probability distribution function of other generators' bids, which is usually translated to a set of scenarios that are equally likely to happen in the electricity market. Each scenario is a set of numerical values for the price bids of other generators. Also, the strategic bidding problem is solved over a time horizon rather than a single time slot, to incorporate the physical constraints that impact the operation of strategic generators over the time [3]. The computation time of the solution method proposed in [4] increases almost linearly as the number of random scenarios or time slots in the strategic bidding problem increases.

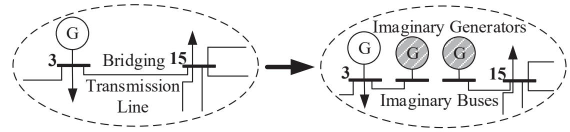

Fig. 3: The transmission line between nodes 3 and 15 in IEEE standard 57 bus system is replaced with strategic imaginary generators and imaginary buses.

IV. Power Grid Partitioning

The computation time of the solution method in [4] increases significantly as size of power grid increases. In this section, the above issue is tackled by deploying a power grid partitioning technique with solution method in [4]. Fig. 2 shows the standard IEEE 57-bus system which is divided into two smaller sub-grids. Each sub-grid includes a subset of nodes in 57-bus system. The transmission lines that fall completely within a single sub-grid are not modified, however the *bridging* transmission lines that connect two nodes in two different sub-grids are replaced with imaginary generators and imaginary buses. For instance, the transmission lines between node 3 and 15 in IEEE standard 57 bus system is a bridging transmission line for the partitioning scheme in Fig. 2, as its terminal points fall in two different sub-grids. Fig. 3 shows how the bridging transmission line between node 3 and 15 is replaced with a couple of imaginary generators and a couple of imaginary buses.

Imaginary generators in Fig. 2 thoroughly model the physical behavior of the bridging transmission lines by enforcing the following equations:

$$|P_{GI,l}| \leq \text{Vector of Bridging Lines' Power Capacities} \tag{40}$$

$$|P_{GI,r}| \leq \text{Vector of Bridging Lines' Power Capacities,} \tag{41}$$

$$P_{GI,l} = -P_{GI,r}, \tag{42}$$

where $P_{GI,l}$ and $P_{GI,r}$ denote the vector of power generation of imaginary generators at left and right sub-grids, respectively. The power generation of an imaginary generator can be positive or negative, but is upper bounded by equations (40) and (41). It worth noting that, the sign and magnitude of an imaginary generator's power generation determines the direction and magnitude of power flow in its corresponding bridging transmission line.

In deploying the partitioning scheme of Fig. 2 along with solution method in [4], the sub-grids in Fig. 2 are regarded as two separate power grids, for which two set of optimization vectors, two set of parameter vectors, and two set of constraints are formulated using the process illustrated in Section II. The following constraints bind the optimization variables corresponding to the left and right sub-grids:

$$\theta_{I,l} = \theta_{I,r}, \tag{43}$$

$$\lambda_{I,l} = \lambda_{I,r}, \tag{44}$$

where $\theta_{I,l}$ and $\theta_{I,r}$ denote the voltage angles of the imaginary buses at left and right sub-grids, respectively. Also, $\lambda_{I,l}$ and $\lambda_{I,r}$ denote the dual variables corresponding to power balance constraints at the imaginary buses located in the left and right sub-grids, respectively.

We denote the vector of optimization variables for the left and right sub-girds by x_l and x_r, respectively. Constraint (40) involves only optimization variables that are stacked in x_l, and therefore can be rewritten in terms of x_l. Similarly, constraint (41) can be rewritten in terms of x_r. However, constraints (42)-(44) involve both of the optimization variables in x_l and x_r, and therefore can be rewritten in the following form:

$$h_{s,l}x_l = h_{s,r}x_r \quad \forall s, \tag{45}$$

where s is an index for imaginary generators and imaginary buses.

Let f_l, F_l, $p_{i,l}$, $p_{i0,l}$, $q_{z,l}$, $q_{z0,l}$, $q'_{z,l}$, $q'_{z0,l}$ and f_r, F_r, $p_{i,r}$, $p_{i0,r}$, $q_{z,r}$, $q_{z0,r}$, $q'_{z,r}$, $q'_{z0,r}$ denote the vector of parameters in the strategic bidding problems for the left and right sub-grids, respectively. The following *convex* optimization problem is the counterpart of problem (35) for the partitioning scheme in Fig. 2, and gives an optimal objective function that is very close to the optimal objective function of problem (1):

$$\begin{aligned}
&\underset{X_l, X_r \in \mathbb{S}^{n+1}}{\textbf{maximize}} \quad trace\left(\begin{bmatrix} 0 & {f_l}^T \\ f_l & F_l \end{bmatrix} X_l + \begin{bmatrix} 0 & {f_r}^T \\ f_r & F_r \end{bmatrix} X_r \right) \\
&\textbf{subject to} \\
&X_{11,l} = 1,\ X_{11,r} = 1 \\
&trace\left(\begin{bmatrix} p_{i0,l} & {p_{i,l}}^T/2 \\ p_{i,l}/2 & \mathbf{0} \end{bmatrix} X_l \right) \geq 0 && \forall i \\
&trace\left(\begin{bmatrix} p_{i0,l} \\ p_{i,l} \end{bmatrix} \begin{bmatrix} p_{j0,l} \\ p_{j,l} \end{bmatrix}^T X_l \right) \geq 0 && \forall i,j \\
&trace\left(\begin{bmatrix} q_{z0,l}q'_{z0,l} & q_{z0,l}{q'_{z,l}}^T \\ q'_{z0,l}q_{z,l} & q_{z,l}{q'_{z,l}}^T \end{bmatrix} X_l \right) = 0 && \forall z \\
&trace\left(\begin{bmatrix} p_{i0,r} & {p_{i,r}}^T/2 \\ p_{i,r}/2 & \mathbf{0} \end{bmatrix} X_r \right) \geq 0 && \forall i \\
&trace\left(\begin{bmatrix} p_{i0,r} \\ p_{i,r} \end{bmatrix} \begin{bmatrix} p_{j0,r} \\ p_{j,r} \end{bmatrix}^T X_r \right) \geq 0 && \forall i,j \\
&trace\left(\begin{bmatrix} q_{z0,r}q'_{z0,r} & q_{z0,r}{q'_{z,r}}^T \\ q'_{z0,r}q_{z,r} & q_{z,r}{q'_{z,r}}^T \end{bmatrix} X_r \right) = 0 && \forall z \\
&X_l \succeq 0,\ X_r \succeq 0 \\
&x_r = \text{Second to Last entries of First Column of } X_r \\
&x_l = \text{Second to Last entries of First Column of } X_l \\
&h_{s,l}x_l = h_{s,r}x_r && \forall s.
\end{aligned}$$

The second to fourth constraints in problem (46) are the counterparts of second to fourth constraints in problem (35) corresponding to left sub-grid. Also, the fifth to seventh constraints in problem (46) are the counterparts of second to fourth constraints in problem (35) corresponding to right sub-grid. The last three constraints in problem (46) bind the optimization vectors of two sub-grids to each other.

On the one hand, the most computation demanding part of the underlying algorithms in semidefinite programming software is the eigenvalue decomposition [5, Section A.5.2] of matrix variables [10], [11]. On the other hand, for the IEEE 57-bus system with the portioning scheme in Fig. 2, problem (46) has two matrix variables of dimension 86×86 and 81×81. Consequently, the computation time of solving problem (46) is mostly dominated by the size of its larger matrix variable, i.e. the size of matrix with dimension 86×86. Since this matrix is of a much smaller size compared to the size of matrix variable with dimension 117×117 in problem (35), problem (46) is solved much faster compared to problem (35), as is shown through numerical studies in Section V.

We note that the imaginary generators should be set *strategic* and be added to the set of generators for the strategic firm, otherwise the SDP solver fails to solve problem (46), see Section V-B. However, the imaginary generators' price bids are not submitted to electricity market, and are included in the analysis only to facilitate solving the strategic bidding problem. Moreover, from equations (42) and (44) we have:

$$P^T_{GI,l}\,\lambda_{I,l} + P^T_{GI,r}\,\lambda_{I,r} = 0, \tag{46}$$

which shows that the profits that are achieved by imaginary generators in left and right sub-grids cancel out each other and therefore don't contribute to the profit of strategic firm. Nevertheless, all the constraints that are imposed on the operation of strategic generators should be imposed on imaginary generators as well.

After solving problem (46), we use the equation (35) to derive the solution vectors $x_l^\star$ and $x_r^\star$ for the left and right sub-grids. The solutions that are given by (35) may not be feasible in problem (1), but can be adjusted according to the process in Section III-B to become feasible in problem (1).

V. Case Studies

A. Simulation Setup

In this section, the performance of the proposed approach in Section IV is studied with respect to the computation time and achieved profit for the strategic firm. The strategic firm is assumed to have two generators that are operating in IEEE standard 57-bus system [12], and are highlighted in color gray in Fig. 2. The reactance of transmission lines in 57 bus system are set according to the data in [13]. The hourly load data is generated by scaling the single hour load data in [13] with a set of scaling factors that result the load profile in [14]. The random scenarios are generated by scaling the price bids of loads and non-strategic generators using the scaling factors 1.1, 1.09, 1.08, 1.07 and 1.06. The price bids of non-strategic generators at bus 3, 6, 8, 9 and 12 are randomly set to 38.4, 35.1, 37, 40.6 and 38.9 \$/MWh, respectively. The hourly price bids for the loads are all set to 72 \$/MWh [4]. The cost function of strategic generators at bus 1 and bus 2 are set to 37.9 and 35.1 \$/MWh, respectively, and the cost function for imaginary generators are set to zero. The capacity of strategic generators are set to 4 and the ramping constraints of

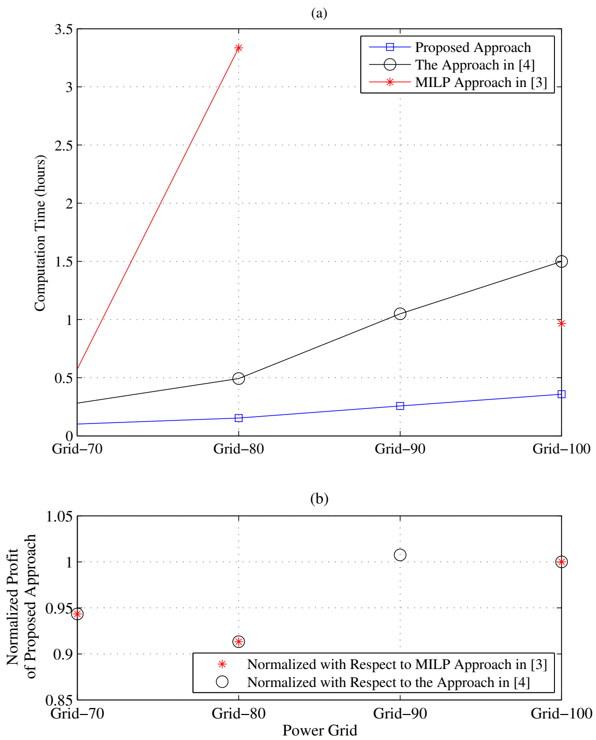

Fig. 4: The impact of the number of loads and generators on: (a) the computation time; (b) the normalized profit.

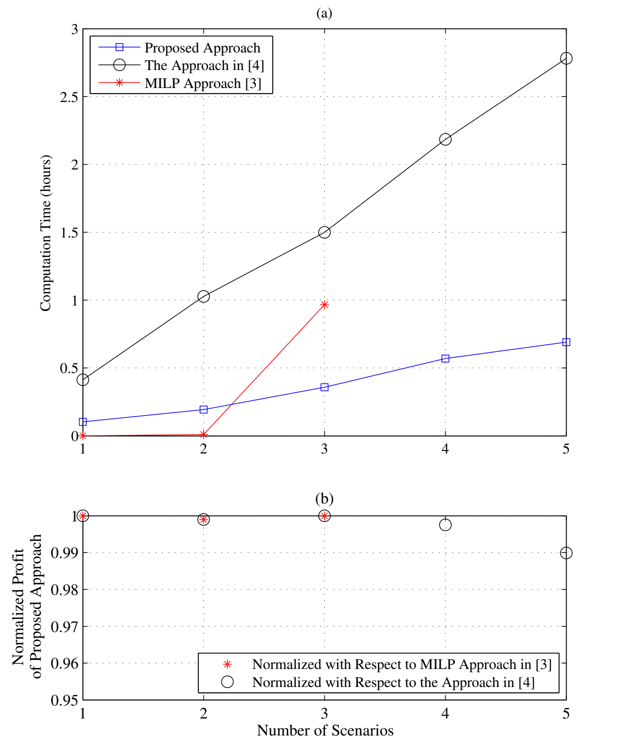

Fig. 5: The impact of number of scenarios on: (a) the computation time; (b) the normalized profit.

generators are set to 0.8. The data for capacity of non-strategic generators is from [12]. The transmission lines from buses 1, 3, 8, 9, 22 and 35, to buses 1, 15, 9, 12, 38 and 36 are assumed to have a capacity of 0.5. The transmission lines from buses 4 and 9 to buses 8 and 12 are assumed to have a capacity of 0.2. Other transmission lines are assumed to have unbounded capacities. Simulations are performed on a personal computer with CPU frequency @2.4GHz and 16Gb RAM, using Yalmip [15] as the optimization modeling software, Mosek [10] as the SDP solver and Gurobi [16] as the MILP solver.

B. Impact of Increasing the Number of Loads and Generators

In this Section, we consider 3 number of random scenarios and 5 time slots in the strategic bidding problem, and study the impact of the number of loads and generators on the computation time of the proposed approach and the approaches in [3] and [4]. We have generated four power grids by randomly removing some of the generators and loads from the IEEE standard 57 bus system, in a way that the total number of loads and generators in the resulted power grids are approximately 70%, 80%, 90% and 100% of the total number of generators and loads in IEEE standard 57-bus system. The resulted power grids are denoted by Grid-70, Grid-80, Grid-90 and Grid-100, respectively, where the Grid-100 is the original standard 57 bus system.

From Fig. 4(b), the computation time of the proposed approach is considerably lower than those of the approaches in [3] and [4]. Also, as the number of generators and loads increase in Grid-70 to Grid-100, the computation time of the proposed approach grows with a slower rate compared to the approaches in [3] and [4]. We emphasize that, the partitioning scheme in Fig. 2 is served as an example portioning scheme to demonstrate the computation advantage of the proposed approach compared to the approaches in [4] and [3]. One may achieve higher computation advantages in using the proposed approach with other partitioning schemes that divide the 57-bus system into smaller sub-grids. We note that, in Fig. 4(a) doesn't include a point for Grid-90 corresponding to MILP approach in [3]. That is because, the MILP approach in [3] doesn't converge for Grid-90 after letting it run for 24 hours.

Also, we note that the computation time for Grid-100 is less than the computation time for Grid-80 when MILP approach in [3] is used. The reason is that, the underlying algorithm in MILP solvers [16] is the branch-and-bound heuristic that doesn't establish a relationship between the computation time and the number of variables in an MILP problem.

Fig. 4(a) shows the achieved profit for the strategic firm normalized by the profit resulted from the approaches in [3] and [4]. From Fig. 4(a), the normalized profit achieved by the proposed approach is always higher than 91% for Gird-70 to Grid-100. Finally, as we emphasized in Section IV the imaginary generators should be set strategic, otherwise the achieved profit from the proposed approach will be poor. For instance, if the imaginary generators in Fig. 2 are set as non-strategic generators, the SDP solver Mosek [10] fails to solve the problem (46) for Gird-70 to Grid-100.

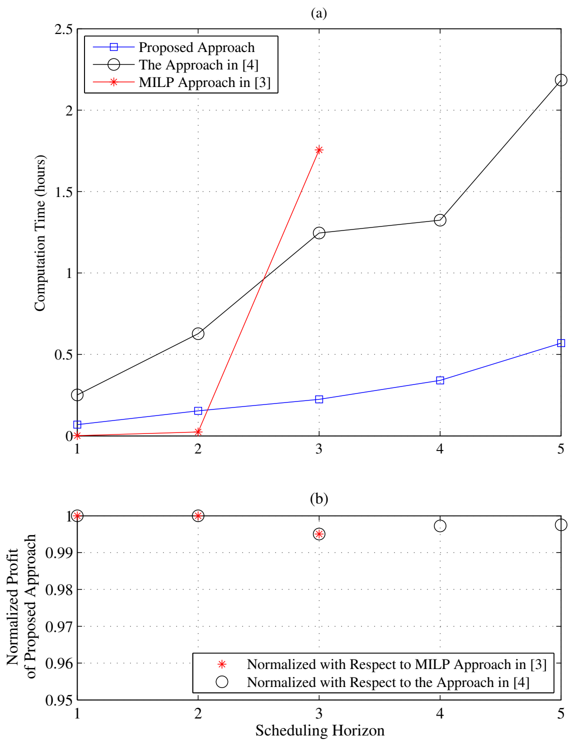

Fig. 6: The impact of the number of time slots on: (a) the computation time; (b) the normalized profit.

C. Impact of Number of Random Scenarios and Time Horizon

As we discussed in Section III-C, in practical cases the strategic bidding problem is formulated for a time horizon and also by considering several random scenarios. Consider the IEEE 57-bus system, i.e. Grid-100 in Section V-B. Fig. 5(a) and Fig. 5(b), respectively, show the computation time and normalized profit for the proposed approach and the approaches in [3] and [4] when four random scenarios are considered in the strategic bidding problem and the number of time slots is increased from one to five. Also, Fig. 6(a) and Fig. 6(b), respectively, show the computation time and normalized profit for the proposed approach and the approaches in [3] and [4] when five times slots are considered in the strategic bidding problem and the number of random scenarios is increased from one to five. From Fig. 5(a) and Fig. 6(a), the proposed approach comes with a considerable lower computational time compared to the approaches in [3] and [4]. Also, From Fig. 5(b) and Fig. 6(b) the normalized profit that is achieved from the proposed approach is always higher then 99%.

VI. Conclusion

In this paper, a partitioning method was proposed that when combined with state-of-art solution method for the strategic bidding problem, results in considerable reduction in the computation time. The proposed approach is scalable in the sense that, its computational time grows almost linearly when the number of elements, i.e. the total number of generators and loads, in a power grid increases. Beside of the computation advantage, the proposed approach is shown to give profit values that are very close to the optimal profit values that one may achieve by obtaining the global optimal solution of strategic bidding problem.

References

[1] H. Haghighat, “Strategic offering under uncertainty in power markets,” *International Journal of Electrical Power & Energy Systems*, vol. 63, pp. 1070–1077, December 2014.

[2] M. Fampa and W. Pimentel, “SDP relaxation for a strategic pricing bilevel problem in electricity markets,” in *Proc. of Simpósio Brasileiro de Pesquisa Operacional*, Natal, Brazil, September 2013.

[3] C. Ruiz and A. J. Conejo, “Pool strategy of a producer with endogenous formation of locational marginal prices,” *IEEE Trans. on Power Systems*, vol. 24, no. 4, pp. 1855–1866, September 2009.

[4] M. Ghamkhari, A. Sadeghi-Mobarakeh, and H. Mohsenian-Rad, “Strategic bidding for producers in nodal electricity markets: A convex relaxation approach,” *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 2324–2336, May 2017.

[5] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2009.

[6] D. C. Lay, *Linear Algebra and its Applications*. Boston, MA: Addison-Wesley, 2000.

[7] S. J. Axler, *Linear algebra done right*. New York, NY, USA: Springer, 1997.

[8] Mathworks, “System of linear equations,” 2016. [Online]. Available: http://www.mathworks.com/help/matlab/math/systems-of-linear-equations.html?refresh=true

[9] L. Vandenberghe and S. Boyd, “Semidefinite programming,” *SIAM Review*, vol. 38, no. 1, pp. 49–95, 1995.

[10] MOSEK ApS, *The MOSEK optimization toolbox for MATLAB manual Version 8.0 (Revision 91)*, 2017. [Online]. Available: http://docs.mosek.com/8.0/toolbox/

[11] J. F. Sturm, “Using SeDuMi 1.02, a matlab toolbox for optimization over symmetric cones,” *Optimization Methods and Software*, vol. 11, no. 1-4, pp. 1029–4937, 1999.

[12] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, “MATPOWER: Steady-state operations, planning and analysis tools for power systems research and education,” *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.

[13] University of Washington, “Power systems test case archive,” 2017. [Online]. Available: http://www.ee.washington.edu/research/pstca/

[14] S. A. Kazarlis, A. G. Bakirtzis, and V. A. Petridis, “Genetic algorithm solution to the unit commitment problem,” *IEEE transactions on power systems*, vol. 11, no. 1, pp. 83–92, 1996.

[15] J. Lofberg, “Yalmip : A toolbox for modeling and optimization in MATLAB,” in *proc. of IEEE International Symposium on Computer Aided Control Systems Design*, Taipei, Taiwan, September 2004.

[16] G. O. Inc., “Gurobi optimizer reference manual,” 2015. [Online]. Available: http://www.gurobi.com